Accounting for Atmospheric motion Vector Error Correlations in the ECMWF 4D-Var and Ensembles of Data Assimilations

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Reasons for the investigations

- Presently in 4DVAR the observation error correlation matrix R is diagonal → observations are assumed to be uncorrelated
- We know that this assumption is not fully correct: Bormann et al. (2003), MWR, showed that SATOB winds are correlated due to e.g. height assignment errors
- To compensate we at the moment apply excessive thinning of data and use inflated observation error
- A proper account for error correlations would allow more correct use of observations and use of more observations

Proposed error correlation method

- Emphasis is on horizontal correlations, but time and vertical correlations are also accounted for in a simple way
- A block diagonal approximation of R for groups of inter-correlated data (same instrument, method, channel)
- Approximation is full-rank for each inter-correlated block
- The implementation is based on a truncated Eigen-vector representation in order to save computational cost
- Based on an algorithm proposed by Mike Fisher, ECMWF (RD Memorandum, R48.3/MF/05106, 2005)

The method explained

$J_o = \mathbf{z}^{\mathrm{T}}\mathbf{z}_{\mathbf{eff}}$ **z** normalized departure; $\mathbf{z}_{\mathrm{eff}}$ the effective norm. departures

$$\mathbf{z}_{\mathbf{i}_{\mathbf{eff}}} = \mathbf{R}_{\mathbf{i}}^{-1}\mathbf{z}_{\mathbf{i}} = \frac{1}{\alpha_i}\mathbf{z}_{\mathbf{i}} + \sum_{k=1}^{N_{RETAIN}} \left(\frac{1}{\lambda_{i,k}} - \frac{1}{\alpha_i} \right) \gamma_{i,k} \mathbf{v}_{\mathbf{i,k}}$$

$$\gamma_{i,k} = \mathbf{v}_{\mathbf{i,k}}^{\mathrm{T}} \mathbf{z}_{\mathbf{i}}$$

Eigen-pairs

Departure coordinate in eigen-space

Parameter accounting for truncation

We can see from this formulation:

1. We don't need the matrix R explicitly available (only its leading "N_{RETAIN}" Eigen-vectors)
2. Two scans through the obs. set are needed:
    1. to compute *gamma* (and of course *Jo* contrib. of uncorrelated data)
    2. to compute the effective departures and thus the *Jo* contrib. of correlated data
3. Eigen-vectors are stored in observation space

The correlation model

- Horizontal correlations introduced as a convolution → multiplication with a weight-function in spectral space
- Going to observation space by interpolation (horizontally, vertically and in time)
- Then the square-root correlation model looks like:

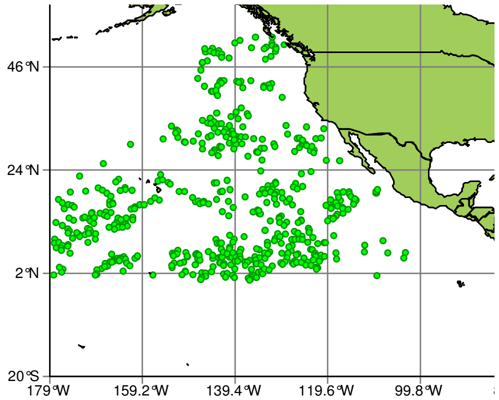
$$U = TS^{-1}D_s^{-1/2}\sqrt{G}$$

$$R = UU^T = TS^{-1}GS^{*-1}T^T$$

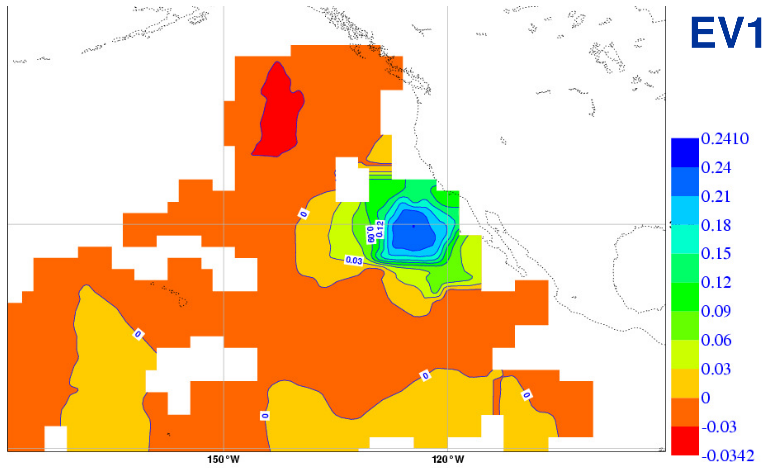
T: interpolator
S: spectral transform (typically T159)
D: spectral inner product weight matrix
G: spectral weight function of the convolution
(Hankel transform of the correlation function)

Implemented method:
Lánczos algorithm to compute the truncated eigen-system of R,
R as sequence of linear operators

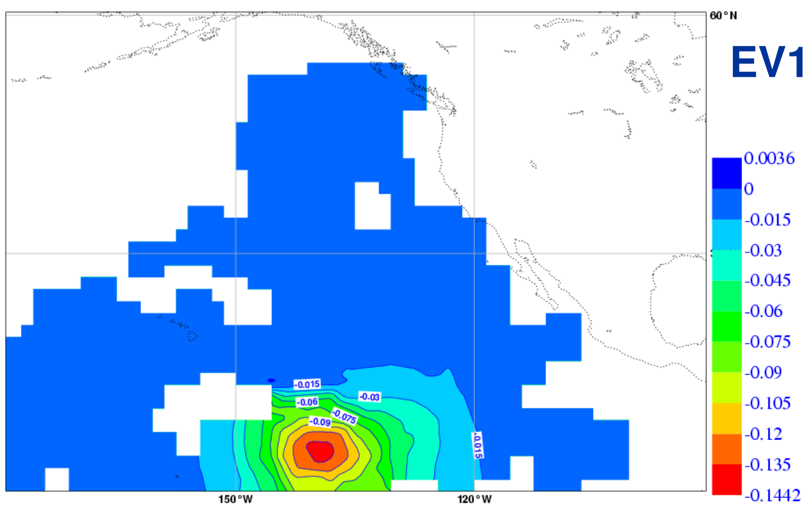
Eigen vectors in observation space

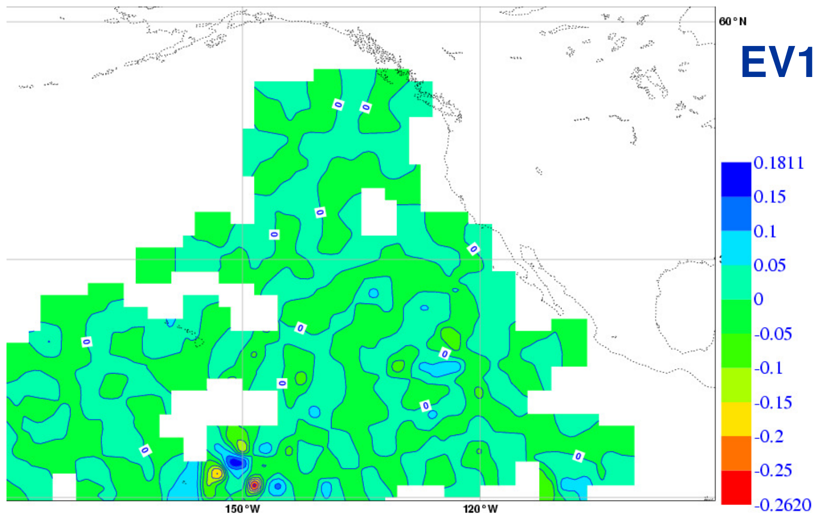


GOES VIS1
850hPa
30 minutes
~500 points



Correlation length scale L=200km





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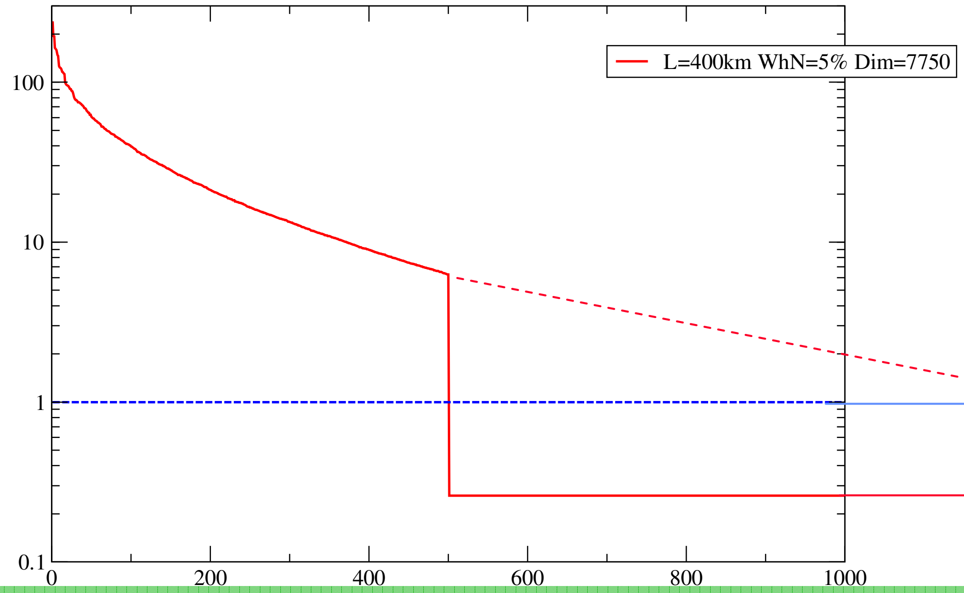
Idealized experiments with perfectly known correlated errors added to radiosonde data

These experiments show the method works.

- The algorithm works well both in terms of degraded analysis fit to observations and improved forecast performance - when error correlations are large enough and perfectly known
- Eigenspectrum of correlation matrix is relatively flat: sufficiently large number of leading Eigen-pairs are needed (typically, when 2000 measurements are correlated, $N_{RETAIN}=50$)
- If errors are uncorrelated and we incorrectly assume a correlated *Jo* → the forecast performance is degraded

Eigen-spectrum of SATOB error correlation matrix

Eigen-spectrum of SATOB error correlation matrix



Jo expressed with the normalized Departures in the Eigen-space of the error correlation matrix:

$$corr.: \quad J_o = z^T \Lambda^{-1} z$$

$$uncorr: J_o = z^T z$$

Spectra of over-penalizing departures with respect to

Ideal, full eigen-representation

Uncorrelated representation

Real experimentation with SATOB winds

A month of 4D-Var, T511/L91.

Three assimilations performed:

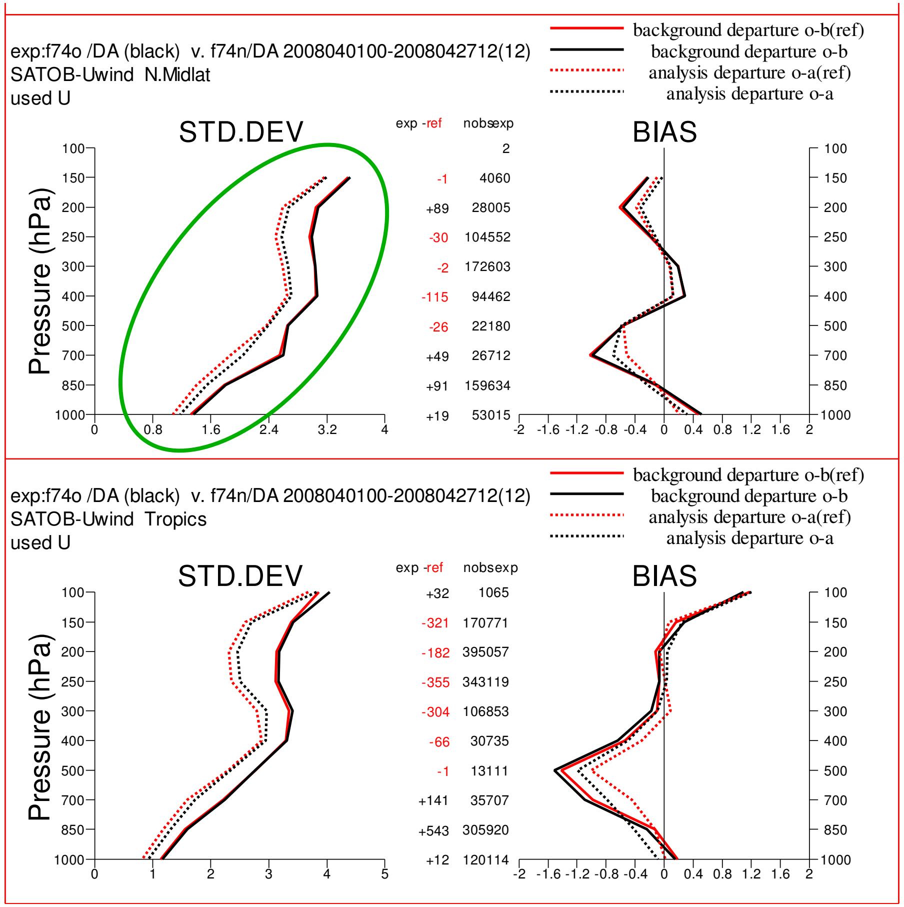
- REF (all obs. assumed uncorrelated)
- Obs corr (assume AMV correlated)
- NoAMV (REF without AMV)

Assumed correlation length scale:
L=200km

Based on Bormann et al., 2003:

$$R(r) = R_0 (1 + r / L) e^{-r/L}$$

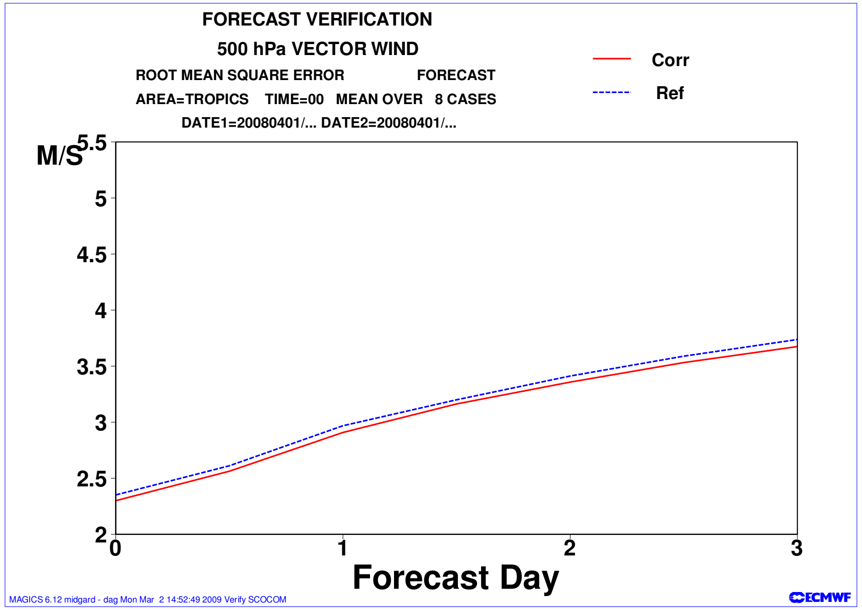
$$R_0 = 0.27 - 0.51; \quad L = 150 - 370 km$$

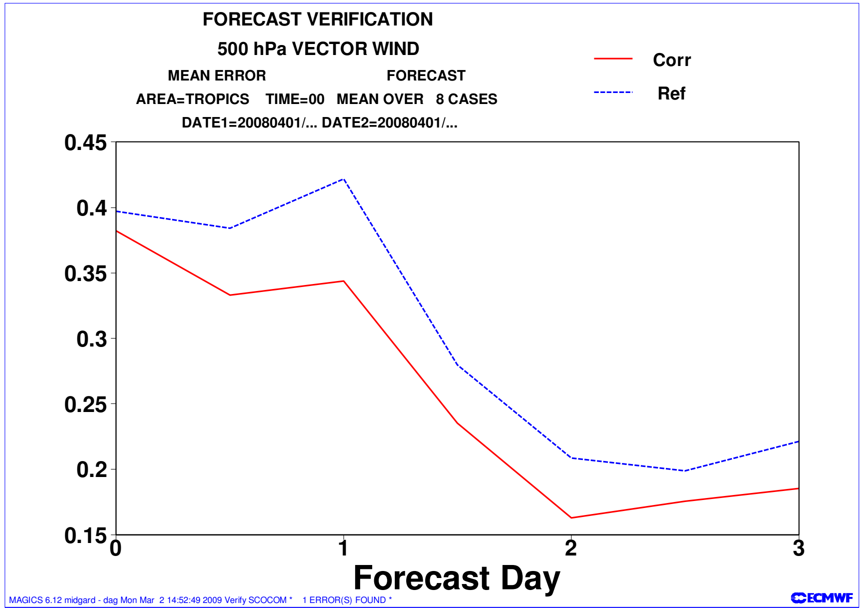


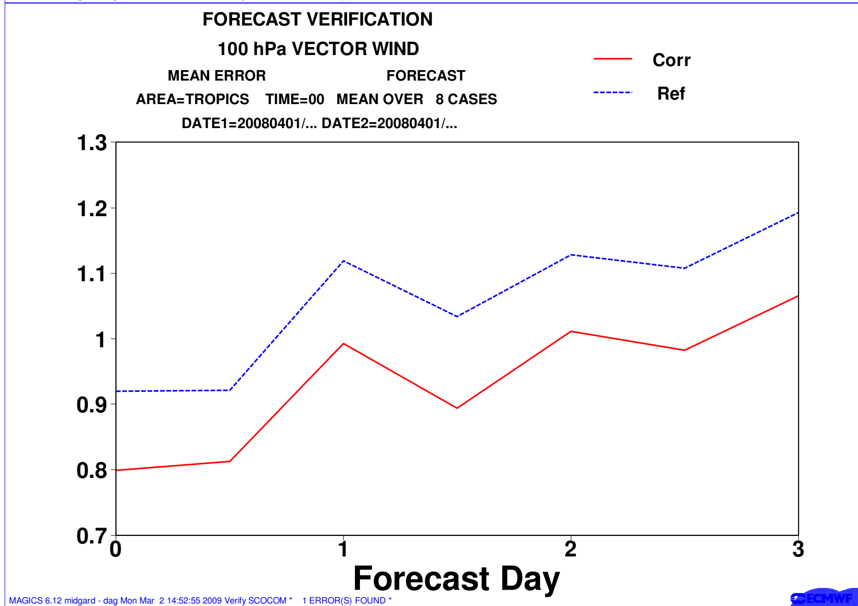
Ref. exp. is red **Obs.corr. exp. is black**

Verified against operational analysis

RMSE



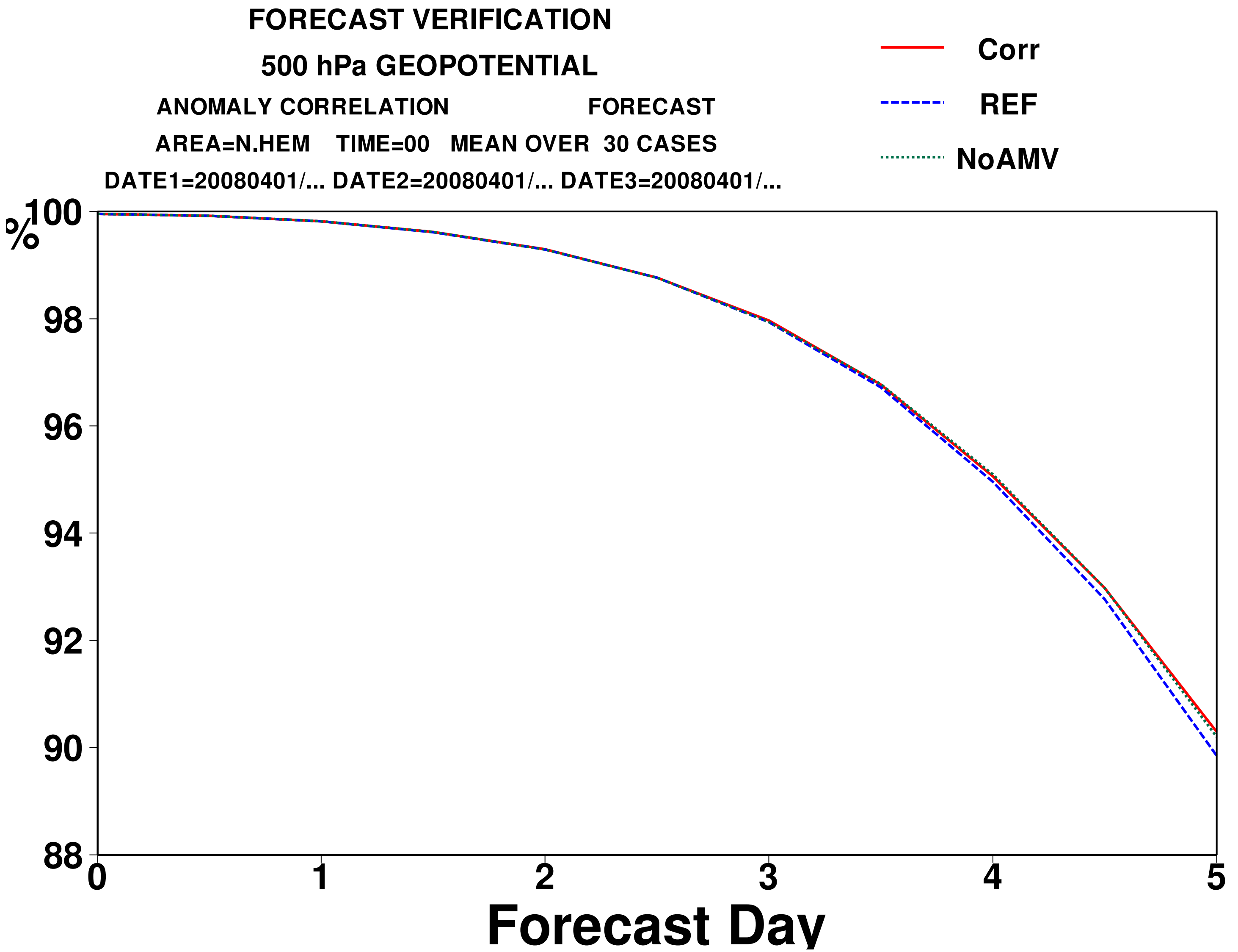




BIAS

Improved RMSE and bias in the tropics where Bormann et al. (2003) found the largest correlation distances, neutral (N.Hem.); slightly negative (S.Hem.) for 8-10 day forecasts

Verified against operational analysis
N.Hemisphere 500hPa geopotential



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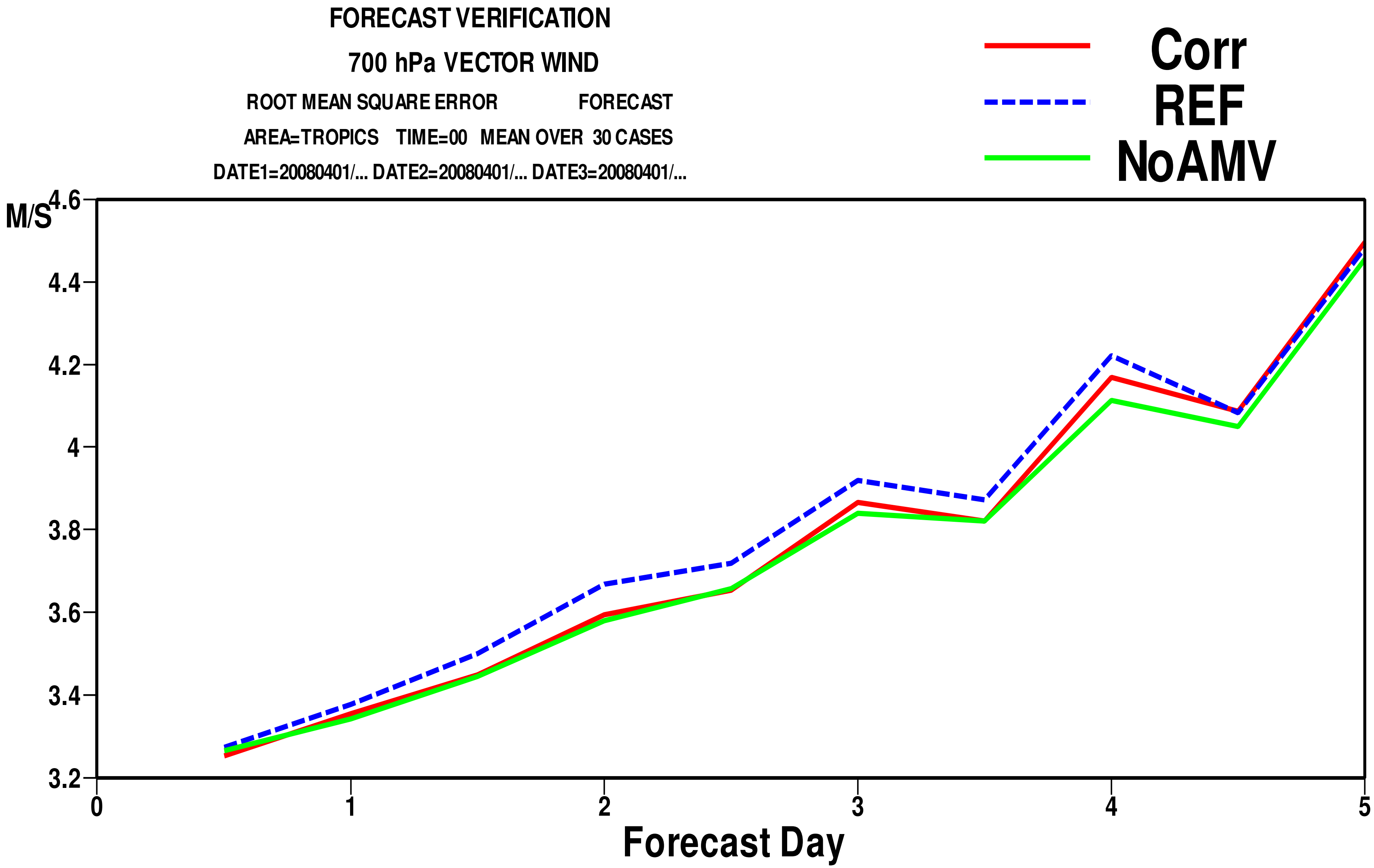
Verification against radiosonde observations



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Summary

- A new method that accounts for horizontally correlated observation errors has been developed at ECMWF
- The truncated Eigen-value decomposition method will be affordable to use operationally
- The results at T511/L91 shows slightly positive results for SATOB winds in the tropics and N.Hemisphere. The results are slightly negative in the S.Hemisphere
- Recent results at T1279/L91 show neutral results
- The same method has been tested for ATOVS data with neutral results
- Further investigation and experimentation required before operational implementation can be envisaged