

Correlation and Relaxation Labelling---An Experimental Investigation on Fast Algorithms

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ABSTRACT

This paper compares experimental results between three popular matching functions, i.e. the Cross-Correlation Coefficient (CCC), the Sum of Squared Difference (SSD), and Sum of the Absolute Value of Difference (SAVD), within our newly developed Correlation-Relaxation (C-R) framework. The C-R framework is a general method for determining optical flow and has been applied to determining cloud motion from satellite images. SSD and SAVD are simpler and faster functions to calculate, when compared with CCC. Combined with a simple Sequential Similarity Detection Algorithm (SSDA), the use of SSD or SAVD can lead to significant savings in computer time in the initial selection of displacement candidates. Given that the image distortion is Gaussian noise, and the motion is translational, the study shows that while computationally expensive, the performance of the CCC function is better, or at least no worse, than using SSD and SAVD in the selection of initial displacement candidates. Similarly, the performance of SSD is better, or no worse, than using SAVD. Computationally, SSD is the fastest among the three functions. In the presence of high level distortion, however, the poor quality of initial candidates selected using SSD and SAVD usually means a large number of iterations of the subsequent relaxation labelling process. In contrast, the CCC function gives high quality initial candidates, and only a small number of iterations are needed. The CCC function also usually leads to better final quality in motion estimation than that produced using the SSD or the SAVD function in the C-R algorithm. In the presence of moderate and low level distortion, however, the performance of SSD is adequate, and its use can lead to faster processing without much sacrifice to the overall motion estimation quality.

1. INTRODUCTION

The recently developed correlation-relaxation labelling (C-R) technique (Wu 1995) is the result of investigating a new methodology for computing cloud motion using sequential imagery from meteorological satellites. It consists of two main components: template matching and relaxation labelling. The template matching procedure identifies initial displacement candidates using a measure such as the Cross-Correlation Coefficient (CCC). The relaxation labelling procedure iteratively refines displacement estimates according to certain constraints such as a requirement that flow fields be smooth.

Cloud motion vector fields are routinely derived by many international meteorological agencies (proceedings of 1st and 2nd Windshop) using template matching based on identifying a maximum cross-correlation coefficient (MCC).

The marriage of cross-correlation matching and relaxation labelling in the C-R technique has significantly improved cloud motion estimation (Wu 1995).

Currently, hourly and half hourly images have been used for cloud tracking purposes. To monitor the atmosphere more effectively, some meteorological agencies, for example, the European organisation EUMETSAT, are now considering using new generation satellites capable of providing images at a time interval as short as 15 minutes. Cloud tracking techniques must therefore be evaluated based on their speed of processing, in addition to the quality of estimation, if they are to be adopted by international agencies. The calculation of numerous correlation coefficients is computationally demanding, as one can see from Equation (1). In our experience, computing CCCs accounts for about 80% of the total computer time used in the C-R procedure.

Simpler functions other than CCC have also been employed widely for matching tasks. Two frequently used functions are the Sum of Squared Difference (SSD), and the Sum of the Absolute Value of Difference (SAVD) (Singh 1991, Barnea and Silverman 1972, Rosenfeld and Kak 1982) where a match is established when the minimum in a SSD (or SAVD) function is identified. In the past, SAVD had the advantage of being computationally efficient as it mainly requires addition operations (Equation (2)). For newer computer architectures, multiplications are done by hardware in a single clock cycle, and it appears that the advantage of SAVD is becoming much less significant. SAVD still uses less computer time than CCC. One can easily see from Equation (3) that SSD also requires less computation than CCC so it is also faster to compute than the CCC. The time needed to compute a minimum SAVD or SSD can be further reduced by invoking the Sequential Similarity Detection Algorithm (SSDA) developed by Barnea and Silverman (1972).

The basic idea behind the SSDA is rather simple. In the process of identifying a minimum dissimilarity measure many dissimilarity measures need not be exhaustively computed. For instance, in the case of determining a minimum SAVD, since it is an accumulated sum of positive numbers, the computation of many SAVDs can be abandoned when their intermediate value exceeds a threshold, or a previous found minimum. The SSDA is not suitable for use maximum similarity measures like the MCC.

In the C-R framework, multiple candidates for a displacement can be selected by identifying a number of highest correlations if CCCs are used, or by determining a number of lowest SAVDs or SSDs if such dissimilarities are used. In our previous work, the CCC has been used. The MCC method is generally expected to perform better than the SSD and SAVD because of its noise suppressing property (Rosenfeld and Kak 1982). This paper, however, investigates the uses of SSD and SAVD within the C-R framework to achieve higher computational efficiency. The objective is to find out whether initial candidate selections using these two functions severely affect the final results obtained by the relaxation labelling process. If the relaxation labelling process can improve the estimates of the SSD or SAVD to a quality comparable with those using the CCC, higher processing speed for C-R algorithm may be achieved.

2. THE COMPUTATION OF CCC, SSD, AND SAVD

For matching between two images $f(x,y)$ and $g(x,y)$, the 2-D CCC function is given by

$$CCC(u, v) = \frac{\sum \sum (f(x, y) - \bar{f})(g(x + u, y + v) - \bar{g}(u, v))}{(\sum \sum (f(x, y) - \bar{f})^2)^{1/2} (\sum \sum (g(x + u, y + v) - \bar{g}(u, v))^2)^{1/2}} \quad (1)$$

where, u and v are coordinates of the displacement space, \bar{f} and $\bar{g}(u, v)$ are mean level corrections of the matching template and window respectively (Wu 1995). Similarly, the 2-D SSD and SAVD functions are

$$\begin{aligned} SSD(u, v) &= \sum \sum ((f(x, y) - \bar{f}) - (g(x + u, y + v) - \bar{g}(u, v)))^2 \\ &= \sum \sum (f(x, y) - g(x + u, y + v) - (\bar{f} - \bar{g}(u, v)))^2 \end{aligned} \quad (2)$$

and

$$\begin{aligned}
 SAVD(u, v) &= \Sigma | (f(x, y) - \bar{f}) - (g(x + u, y + v) - \bar{g}(u, v)) | \\
 &= \Sigma | (f(x, y) - g(x + u, y + v) - (\bar{f} - \bar{g}(u, v))) |
 \end{aligned} \tag{3}$$

Readers are referred to our earlier publication (Wu 1995) for detailed discussions on these functions.

To appreciate the speed of these matching functions a simple software program was written to determine the time consumption of addition, multiplication, assignment, relational operation, squaring, and absolute value operation using our IBM RISC6000 workstation. It was found that, on average, addition, multiplication, squaring operation, assignment, and relational operation use a similar amount of time. However, the absolute value takes almost twice as much time since it has to be seen as the combination of a relational operation and an assignment operation. If one defines the time used for an addition as one time unit, the SSD requires three units, two additions and one squaring, before summation as seen in (2). From (3) it is seen that SAVD requires four such units, two for addition, one for relational binary test, and one for assignment, before summation. This explains why SSD is faster than SAVD in modern computers. For the CCC function, Equation (1) shows that the computation consists two parts: the summation for cross-correlation in numerator and the summation for the variance in the denominator. Inside the first summation, three time units are used, two units for addition, one for multiplications. Inside the second summation one unit for addition and one for squaring are needed. The CCC is therefore slower than SSD and SAVD, and this is in agreement with our experimental results. Note that the term $\Sigma \Sigma (f(x,y) - \bar{f})^2$ in (1) is not considered in the above discussion. This is because it is independent of u and v so that its computation is insignificant compared to other terms. Also, in (2) and (3), $(\bar{f} - \bar{g}(u, v))$ is a constant that can be obtained before the cumulative summation. The above discussion also indicates that the use of FFT for computing CCC may not have any advantage at all over direct evaluation when multiplications can be done in hardware.

In using SSD and SAVD in the C-R procedure, we adapted the use of SSDA for selecting a number of lowest minimum measures.

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

An image sequence from the Japanese geostationary meteorological satellite (GMS) has been used for this work. In this paper, the three matching functions are first applied to simulated motion image pairs under controlled conditions to examine the behaviour of the these functions. They are then applied to a GMS cloud motion image pair, and the results are examined. The simulation experiments provide insight into understanding the results of real motion tracking.

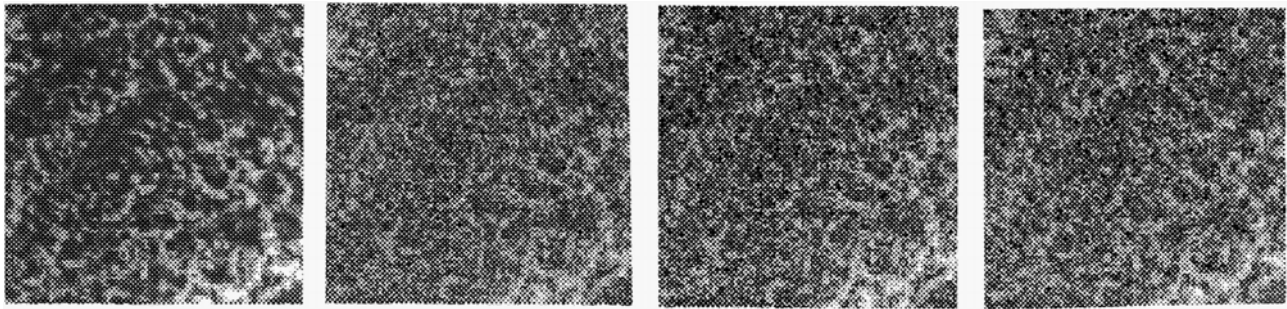
3.1 Simulated Motion

Simulation studies were performed using image pairs with known translational motion and distorted by computer generated Gaussian random numbers. The first images of all simulated motion image pairs are identical, as illustrated in Fig. 1(a). It is extracted from a large GMS image covering an area of shower clouds.

The image pixel grey levels in Fig. 1(a) have a standard deviation of 12.48. To add known levels of distortion to the image, 2-D zero mean Gaussian noise images of standard deviation (sd) ranging from 1.0 to 12.0 were created, at sd increments of 1.0. Each of these noise images was then added to a coordinates translated version of the first image to generate a distorted motion image for use as the second of a motion image pair.

Three case studies are presented in this paper. In *case 1*, the second simulated motion image, as shown in Fig. 1(b), was generated in two steps: first the sub scene was extracted from the same original GMS image, but shifted down and left 3 by 3 pixels; then, a noise image with a standard deviation of 12.0, or 96% of the standard deviation of the image signal, was added to the extracted sub image. The simulated uniform motion vector is therefore (3,3) in pixels. In *case 2*, the second simulated motion image, as shown in Fig. 1(c), was generated in the same manner as Fig. 1(b), but the Gaussian noise has a standard deviation 11.0, or 88% of the standard deviation of the image signal.

In *case 3*, the second simulated motion image, as shown in Fig. 1(d), was also generated in the same manner as Fig. 1(b), and with a Gaussian noise level 88% of that of the image signal. However, different seeds were used for generating random numbers to see whether consistent result can be obtained.



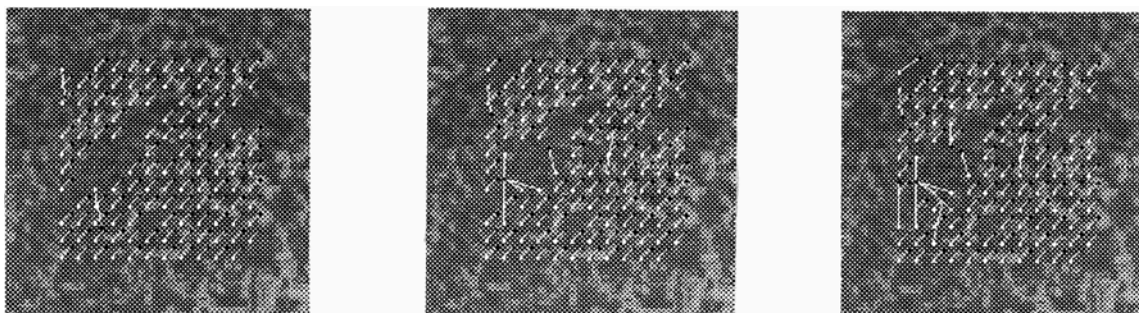
a) Original b) 96% noise (Case1) c) 88% noise (Case2) d) 88% noise (Case3)

Figure 1. Images that make up the three motion image pairs in simulation studies: a) and b); a) and c); and a) and d). The three image pairs correspond to the three cases presented in this paper. Note that in Case 3 the seed used to generate the random numbers is different from that in Case 2.

Simulations were also done using distortion level at 75% or less of the signal standard deviation. However, the correct results were always obtained using any one of CCC, SSD and SAVD without relaxation labelling processing.

High Level Distortion (Case 1):

Fig. 2(a), (b), and (c) are the results of the MCC, the minimum SSD, and the minimum SAVD respectively, i.e. the results before relaxation labelling. Of all the 123 points (the white dots which indicate the starting points of vectors) whose displacements are estimated, only 2 erroneous estimates are found in Fig. 2(a), while 7 are in Fig. 2(b) and 11 are in Fig. 2(c). This is consistent with the expectation that the MCC performs better than the minimum SSD and SAVD. In this example, the CPU times used by the MCC, SAVD and SSD are, respectively, 45 seconds, 32 seconds, and 23 seconds. Therefore, the changeover from MCC to SAVD leads to a 29% time savings, and that from MCC to SSD leads to a 50% saving.



a) MCC b) SSD c) SAVD

Figure 2. Results of initial displacement estimation from the three matching functions in simulation studies, i.e. the results before relaxation labelling: a) MCC; b) SSD; and c) SAVD.

In all examples given in this paper involving relaxation labelling, the number of candidate displacements is $n = 10$. We are interested in finding answers to the following questions:

- 1) How well do these three functions, i.e. SAVD, SSD, and CCC, select candidate displacements?
- 2) How does the initial selection affect the performance of the relaxation labelling?
- 3) What is the time use to achieve error free results corresponding to each function.

To answer the first question, we examine the initial displacement candidates of all the 123 points, and in particular, those points for which erroneous estimates are obtained before relaxation labelling processing. In the case of CCC, all true displacements are selected as candidates, and for the two erroneous estimate points, the correct

displacements rank No. 2 and No. 4, in terms of their initial likelihoods estimated. For SSD, all true displacements are also selected as candidates. Among the 7 erroneous estimation points, 5 rank No. 2, 1 ranks No. 3, and 1 ranks No.4. In the case of the SAVD, all true displacements are included as candidates as well. However, of the 11 erroneous estimate points, some the true displacements' rankings are rather poor, with 6 at No 2, 1 at No. 3, 1 at No. 5, 2 at No. 6, and 1 at No. 8. Candidates which are not true displacements have also been examined. Overall, the CCC selected more candidates which were close to the true displacements, while the SAVD identified many candidates with higher deviations from the true displacements. The performance of SSD is somewhere in between. This is not surprising since the cross-correlation function is relatively insensitive to noise (Rosenfeld and Kak 1992). The results may also be due to CCC and SSD being second order statistical matching functions and therefore more suitable for the Gaussian noise we added.

In running the relaxation labelling procedure, it is found that only 1 iteration is needed to achieve an 100% correct displacements using the CCC selected candidates, while 20 and 50 iterations are needed using those of the SSD and SAVD respectively. Fig. 3 illustrates the results of relaxation labelling using the candidates of SSD after 2, 8, and 18 iterations. Similarly, Fig. 4 shows the results of relaxation labelling using the candidates of SAVD after 4, 16, and 48 iterations of processing. These results show that in the early iterations the relaxation labelling does not necessarily improve the estimation due to poor initial candidate selection. In fact, it can make it worse, as is shown in this example. This is because the relaxation labelling process uses all candidates, whether or not they are true displacements, as constraints, and a false displacement in a candidate list may also be selected as the estimate. This is especially true when many high initial likelihood candidates deviate significantly from the true displacements. However, because the majority of true displacements have the highest likelihood ranking in their respective candidate lists, the processing does eventually converge towards the correct result. The relaxation processing on the candidates of SSD behaves similarly to that for SAVD but uses fewer iterations due to its relatively higher quality of initial labelling. In the case of the CCC, however, the high quality initial candidate selection enables the system to converge very quickly.

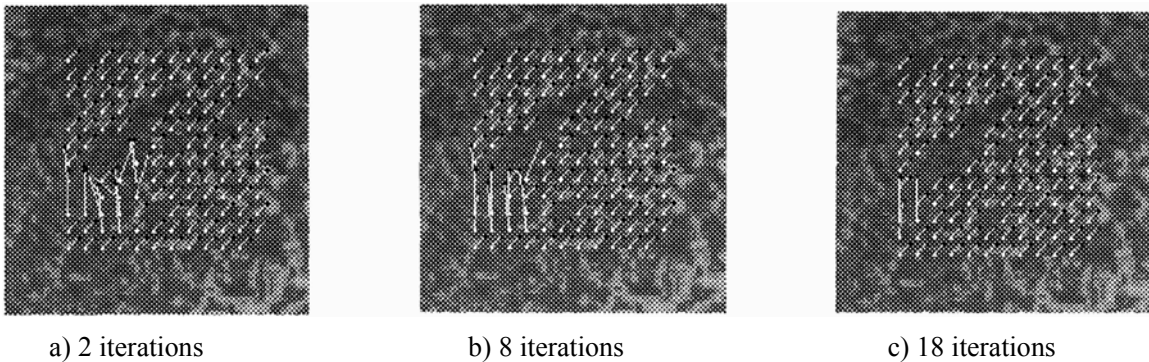


Figure 3. The result of relaxation labelling applied to displacement candidates selected by SSD in simulation studies: a)-c) 2, 8, and 18 iterations. A total of 20 iterations, or 56 seconds of CPU time, are needed to achieve error free results. See Section 3.1 for details.

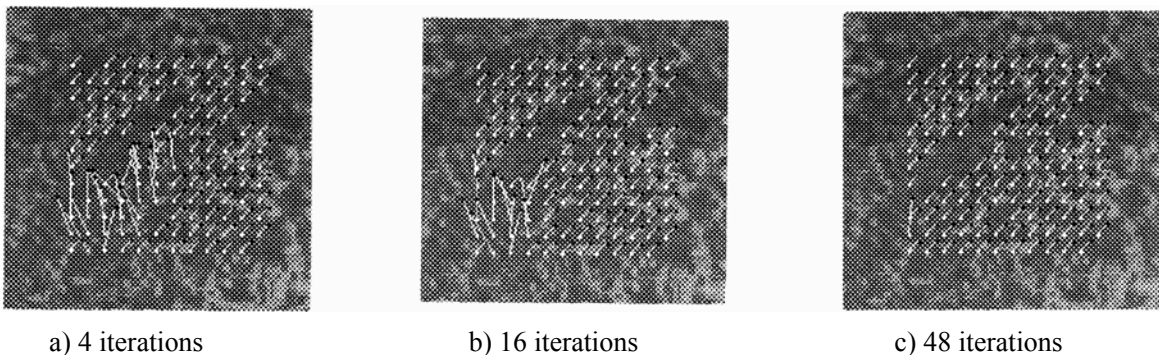


Figure 4. The result of relaxation labelling applied to displacement candidates selected by SAVD in simulation studies: a)-c) 4, 16, and 48 iterations. A total of 50 iterations, or 92 seconds of CPU time, are needed in this case to achieve error free results. See Section 3.1 for details.

The CPU time taken for relaxation labelling processing depends on the number of iterations. However, later iterations usually require less time to perform since some candidates are deleted whenever their updated likelihoods become lower than a certain threshold. In above examples, the first iteration used 4 seconds CPU time. In the case of SSD, 20 iterations used a total of 56 seconds. For SAVD, the 50 iterations used 92 seconds. Therefore, in this experiment, the C-R algorithm employing SAVD causes the overall time of processing to be much longer than that required when invoking the CCC function. Although the C-R algorithm invoking SSD is a lot faster than that using SAVD, the relaxation time is still significantly longer than that for CCC.

Moderate (Cases 2 and 3) and Low Level Distortion (75% or less):

In the cases of moderate to lower distortion levels, results consistent with those in Case 1 have been observed regarding the relative performances of the three matching functions in selecting initial candidates. Because of lower distortion levels, the initial labelling qualities of the three matching functions in both Cases 2 and 3 are superior to those of their corresponding results in Case 1. Consequently, the number of iterations of relaxation labelling processing needed to achieve 100% correct results are substantially reduced. The numerical values indicating the performances and the computation time for all three cases are given in Table 1 for comparison.

Experiments using noise levels 75%, or lower, that of the signal deviation show that both SSD and SAVD can replace CCC to select displacements without relaxation labelling, since no error has occurred. Although it cannot be very exhaustive, it is believed that the likelihood of getting poor estimates, when noise level is low, is very small. Under the moderate distortion condition, all results show that the performance of SSD is superior to that of SAVD, in addition to being much faster to compute. The results in Table 1 indicate that, when distortion level is still high but lower than that in Case 1, SSD should be considered a good alternative for the CCC function for two reasons. The first is that it resulted in relative high quality initial displacement candidates, and the second is its fast computational speed.

Table 1.

Noise Level (standard deviation as percentage of signal deviation)	Matching Function used in correction matching	CPU time used for initial correlation matching (seconds)	Number of erroneous estimates in correlation matching	Number of iterations of labelling for achieving 100% correct results	CPU time used for relaxation labelling (seconds)	Total CPU time	
						In seconds	relative
96% (Case 1)	SAVD	32	11	50	92	124	2.8
	SSD	23	7	20	56	79	1.8
	CCC	45	2	1	4	49	1.1
88% (Case 2)	SAVD	32	5	26	69	101	2.2
	SSD	23	3	8	29	52	1.2
	CCC	45	0	0	0	45	*1.0
88% **(Case 3)	SAVD	32	3	6	23	55	1.2
	SSD	23	0	0	0	23	0.5
	CCC	46	0	0	0	46	1.0

*45 seconds CPU time is used as the basis for determining relative times.

**different seeds are used for generating random number in this case from Case 2.

3.2 Real Motion

The identical background cloud image in Fig. 5 shows the first of a pair of one hour lapsed GMS images. Computer animation shows that the shower clouds move rather uniformly towards the top-right corner. Fig. 5(a), (c) and (e) show the results of MCC, SSD and SAVD respectively. It can be seen that while all three results contain spurious estimates, the MCC result is marginally better than those of the SSD and SAVD in terms of flow field coherency.

The performance of the relaxation labelling on Fig. 5(a) is demonstrated in Fig. 5(b), where 3 iterations of labelling are processed. Similarly, Fig. 5(d) shows the result of 12 iterations of labelling processing on Fig. 5(c), and Fig.5(f) the results of 24 iterations on Fig. 5(e). It is noted that more iterations were performed in each case, but the results did not show significant changes. While relaxation labelling has improved the estimation in all cases, it is apparent that the vector field in Fig. 5(b) is still more coherent than those in Fig. 5(d) and (f), indicating that the final quality of estimation using the CCC function is better than that using the SAVD and SSD. The resultant qualities of the SSD and SAVD appear to be similar in this case. However, the result of the SSD requires only half the computation time used for that of the SAVD. The relative CPU time consumption in initial candidate selection and relaxation labelling using the three functions follow patterns similar to those discussed in the simulation studies. In our experience with GMS images, most processing using the CCC function takes only 3 or 4 iterations.

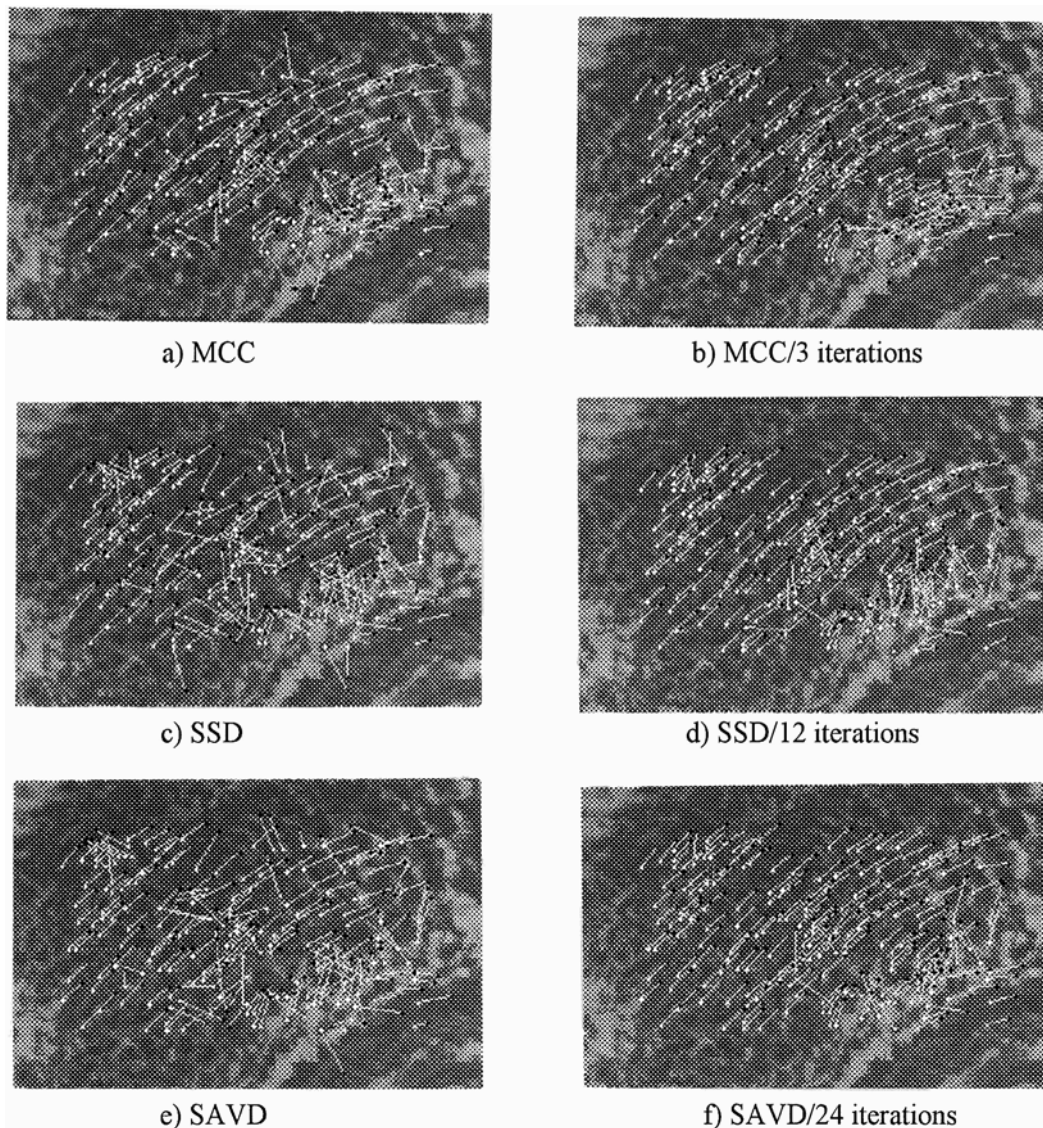


Figure 5. Motion estimation from a GMS cloud image pair. a) and b): The results of MCC and that after 3 iterations of relaxation labelling; c) and d) The results of SSD and that after 12 iterations of labelling; e) and f) The results of SAVD and that after 24 iterations of labelling. Section 3.2 discusses in detail these results.

4. SUMMARY

Experiments have been conducted using images with simulated motion and image distortion, as well as real GMS cloud motion images, for investigating the uses of faster matching functions SSD and SAVD in the correlation-relaxation framework. Our investigation leads to following observations:

- 1) Matching functions SSD and SAVD are significantly faster than the CCC, with the SSD being the fastest of the three. The use of SSD for initial candidate selection reduces computer time by as much as 50% compared to the CCC. The use of SAVD reduces the time by about 29%.
- 2) In general, the CCC performs better than the SSD and SAVD in selecting the initial displacement candidates in the presence of image distortion. Similarly, the SSD performs better than the SAVD.
- 3) The relaxation labelling processing is sensitive to the quality of initial candidate selection.
- 4) In the presence of low level distortion both SSD and SAVD can perform adequately. Their uses in replacing the CCC can achieve significant computational savings.
- 5) In the case of moderate level distortion, the performance of the SSD is adequate, although it may still be poorer than that of the CCC for use in the C-R technique, since it can lead to significant reduction in the overall processing time. Under similar conditions, the performance of the SAVD is still very poor, and a large number of relaxation iterations is usually required.
- 6) In the case of high distortion level, the high quality of displacement candidates resulting from the CCC ensures that only a small number of iterations are needed in the subsequent relaxation labelling process. Under the same condition, both the SSD and SAVD perform poorly. A large number of iterations of relaxation labelling is usually required to improve their estimation. Their uses cause the overall processing time to be much longer than that needed when the CCC is used in the C-R technique. This is particularly true when SAVD is used.

The above observations indicate that in the presence of high level image distortion, the CCC appears to be the best option for C-R technique. In the cases of low to moderate level distortions, both SSD and SAVD may be used as alternatives for the CCC. However, SSD appears to be a much better alternative than SAVD for the CCC function, not only because it is faster to compute but also due to its better quality in initial candidate selection.

The initial experiments are conducted under two conditions: a) translational motion and b) Gaussian noise distortion. The three matching functions are inherently suitable for translational motion. Their tolerance towards rotation is not well understood. More experimental evaluation is needed, with regard to the levels and types of image distortion, as well as motion characteristics, to achieve an in depth understanding of the performances of matching functions.

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