NONLINEAR OBJECTIVE ANALYSIS OF WIND FIELD FOR NWP MODELS

Kunhikrishnan, T and @S.Rajamani

Agricultural Meteorology Division, India Meteorological Department, Shivaji Nagar, Pune-411005, India

ABSTRACT

The present study is to formulate and test the objective analysis of wind field subject to nonlinear balance equation as constraint. The analysis scheme covers the role of initialization and nonlinearity in the analysis. The analysis problem is nothing but an inversion technique of the form $V = K_n^{-1}(\nabla^2\Psi)$ where Ψ is the stream function, V is the wind fleld and K_n is the nonlinear operator. The wind field can be obtained from the equation $V_{n+1} = V_n + QW\ (z-z\ (V_n))$ by analysis correction method, where Q is the normalization factor which represent data density and W is the weight to grid point which can be modelled by a continuous covariance function.

The scheme has been tested by taking August 1988 Monsoon depression for five days. It is found that they depicted the circulation features and the centre of depression fairly well. Comparing RMS error with other schemes, nonlinear scheme is found to be good. The wind field which is more important in the tropics is analysed with the constraint of nonlinear balance equation rather than geostrophic balance. The preliminary studies indicate that the analysis result is encouraging, but further experiments are necessary. The analysis scheme could be implemented in models and has the potential to improve the forecast in the tropics.

@Forecasting Research Division, Indian Institute of Tropical Meteorology, Dr. Homi Bhabha Road, Pune-411008, India

1. INTRODUCTION

Many of the practical problems in objective analysis are not linear. Use of data which are nonlinearly related to the meteorological parameter to be analysed is often encountered e.g. use of radiance data in the analysis of temperature. Tropical region is data sparse region and hence cause a lot of uncertainty in the analyses. However the advent of satellite technology and subsequent increase in space and time coverage of observations has provided a viable means of improving the accuracy and range of Numerical forecast.

Charney et al. (1969) suggested that combination of background and current data in a numerical model such that model equation provide time continuity and dynamic coupling. In addition to observations, climatological data, prior forecast valid at analysis time are often expressed as input to Objective Analysis.

1.1 Linear analysis scheme

Most of the methods of objective analysis schemes are considered as linear transformations. The representation is characterised by transformation of a set of variables (X) into a set of one or more output variables (Y) through a linear operation. According to Mcpherson (1976) objective analysis scheme in both the methods viz. grid point as well as spectral take the linear forms as :

$$X_a = X_b + \Sigma W\ (Y_o - Y_b)$$ where w is the weight

$$X_a = X + \Sigma aifi$$ where f = orthogonal basis function

In the linear objective analysis procedure the weights are independent of the actual observed values, but depend only on their positions and accuracies. Moreover it is assumed that the error distributions are Gaussian and the prior constraints and relationship describing the desired analysis are linear. In case of linear variational problems, the penalty function must be quadratic and analytic solution should exist. Among most of the linear analysis scheme optimum interpolation (OI) scheme is widely used in assimilation procedures for large scale prediction models (Lorenc 1981, Hollingsworth 1986, Mcpherson 1975). The OI method was introduced by Eliassen (1954) and Gandin (1963). The analysis technique is based on statistical linear regression and provides a systematic framework for blending observations of different error characteristics with recent prediction or climatology. In India, Rajamani et al. (1983) developed the OI scheme for wind field at all standard levels and has incorporated the divergent part in order to reduce the spin up problem in the tropics.

1.2 Nonlinear Analysis scheme

The data which are nonlinearly related to the analysis parameter is a crucial problem for operational assimilation scheme. OI analysis scheme is linear and fails to extract properly from the observations of any quantity linked to the forecast model variable by a nonlinear link. This restricts the use of indirect observations (e.g. : Radiance data from satellite) and many future observing systems (scatter meter observations) and current conventional data. Nonlinear optimum analysis scheme suggested by Lorenc (1981, 1986, 1988a, 1981) gives a new approach for solving such problems.

In this work, it is attempted following Lorenc to study the nonlinear scheme which is applicable to Indian region. Here we have modified the scheme which would be suitable for Asian monsoon conditions. For this we have used the observed geopotential height for the analysis of wind field through nonlinear balance equation. It partially covers the role of initialization and nonlinearity in the analysis. According to Lorenc, the analysis procedure can be applicable for 4D analysis involving time dimension, analysis of near discontinuties, multivariate analysis subject to non linear constraints and indirect relationship between observed parameter and analysed. The method can be used for the inversion of satellite sounding radiances for the analyses of vertical temperature and humidity profiles (Eyre, 1989). In this case the link operator is radiative transfer equation.

The nonlinear scheme is nothing but a variational scheme consists in minimising a cost function by using an iterative method which requires the computation of cost functions and its gradient at each minimisation steps. The analysis scheme incorporates non linear prior constraints in the analysis and observations having non Guassion error distribution.

2. METHODOLOGY

2.1 Basic equation for Nonlinear Optimum Analysis scheme

According to Lorenc (1986, 1988) the analysis is nothing but an inversion technique of the form $X = k_n^{-1}(Y)$ where k_n is non linear operator, X be a discrete basis for its model state and Y be the observed vector.

It X is a simple field, Y is a set of observations of that field, then k_n is simply an interpolation from that field to the observation positions. In case of indirect observations k_n represent the model equation connecting X and Y. If X is the initial state for a model, and Y is a set of all observations over a period of time, then k_n is a model forecast upto the time of observations.

The variational equation for the best analysis is derived from a general Bayesian equation expressed in terms of multidimensional probability distribution function. The background and observational information combined in Bayesian minimum variance calculation which minimises a functional of the unknown solution. We also assume that neither the analysis scheme nor the observation space are full representation of the true atmosphere so that in general k_n will have the resentation error. Common error due to representivity (F) is connected with observation error (O+F).

The maximum likelihood estimate of the best analysis X_a is defined as that 'X" which maximizes the probability distribution function (p.d.f.) $P_a(X)$.

$$\text{where} \quad P_a(x) = \frac{P_{OF}\,[Y_o - k_n(x)]\ P_b\,(X - X_b)}{\int P_{OF}\,[Y_o - k_n(x)]\ P_b\,(X - X_b)\,dX}$$

$P_{OF}\,[Y_o-k_n(X)]$ representing the error of representivity and observation errors. $P_b[X-X_b)$ represent the P.d.f which describes the background error.

Maximising $P_a(X)$ is same as maximising $P_{OF}\,(Y_o-k_n(X)\ .\ P_b[X-X_b)$.

$$\text{where} \quad P_{OF}(Y_o-k_n(X).P_b(X-X_b).=\text{Exp}[-\tfrac{1}{2}(Y_o-K_n(x)]^T(O+F)^{-1}$$
$$(Y_o-k_n(X)]-[(X-X_b)^T\ B^{-1}(X_b)]$$

Maximising $P_a(X)$ is equivalent to minimising a penalty function

$$J(x) = -\log_e\{P_{OF}(Y_o-K_n(X)] - P_b(X-X_b)\}$$
$$= [\ Y_o - K_n(X)\ ]^T\ (O+F)^{-1}\ [(Y_o - K_n(x)] +$$
$$(X - X_b)^T\ B^{-1}\ (X-X_b)$$

where O,F and B represent the error covariance matrix for observation, representation and background respectively.

$$O = \langle (Y_o - Y_t)\ (Y_o - Y_t)^T) \rangle$$

$$F = \langle [Y_t - k_n(X_t)]^T\ (Y_t - k_n(X_t \rangle]$$

$$B = ([X_b - X_t)\ (X_b - X_t)^T)\ \langle (X_b - X_t)\ (X_b - X_t)^T \rangle$$

2.2 Solution for Analysis equation

The analysis problem is the minimization of a penalty function by using Gauss-Newton Iterative method by using Descent Algorithm (Gill et al. 1981). The simplest algorithm is to go from the current best estimate X_n to a new estimate X_{n+1} in the direction of steepest descent indicated by $J'(x)$.

The iterations are given by

$$X_{n+1} = x_n - \frac{J'}{J''}$$

where $J'(X) = -2\ [K^T\ (O+F)^{-1}\ [Y_o - k_n(x))] + B^{-1}\ (X - X_b]$

K = Jacobian Matrix of k_n

$$J''\ [x] = 2\ [K^T\ (O+F)^{-1}\ K + B^{-1}]$$

The successive correction scheme for the practical iteration procedure was proposed by Bratseth (1986).

$X_{n+1} = X_n - (J'')^{-1}\ J'$ It is given by

$$X_{n+1} = X_n + QW\ [Y_o - K\ (X_n)]$$

where $W = BK^T\ [O+F]^{-1}$ gives the weight which depends on the distance between observations and grid points and on the observational errors. Q is considered as local data density function at each grid point which can be approximated by the reciprocal of sum of weights at each grid point.

$$Q = (wk + 1)^{-1}$$

Convergence of iteration depends upon the initial guess (Background)

2.3 Nonlinear optimum analysis for wind field

Lorenc analysis scheme can be applicable for the analysis of wind field subject to nonlinear balance equation as weak constraint to the atmosphere. We have formulated and tested the scheme for objective analysis of wind field over Indian region and adjoining areas. The analysis problem is nothing but an inversion technique of the form

$$(u,v) = k_n^{-1} (\nabla^2\Psi) \text{ where}$$

$$\nabla^2\Psi = f\xi + \beta u - 2 J(u,v) \quad \nabla^2\Phi = \nabla(f\nabla\Psi) + 2 J(u,v)$$

The stream function (Ψ) can be obtained through nonlinear balance equation using geopotential (Φ) as input.

The vectorial form of the balance equation is given by

$$\nabla^2\Psi = \nabla(f\nabla\Psi) + 2 J(u,v)$$

where ∇ and ∇^2 are gradient and Laplacian operators respectively. J is the Jacobian term. f is the coriolis parameter. k_n represent the nonlinear link between geopotential field and (u, v).

Our analysis problem is to find (u,v) from Z-field by successive approximation

The analysis equation for V = (u,v) is given by

$$J(u,v) = J(V) = (Z_o-Z_E)^T(O+F)^{-1}(Z_o-Z_E)+(V_b-V)^TB^{-1}(V_b-V)$$

For the maximum likelihood estimate of (u,v) the penalty function J(u,v) should be minimum. The minimization can be done by iteration procedure

$$V_{i+1} = V_i + QW\,[Z_o-k_n(V_i)]$$

where k_n (V_i) is the estimated value of Z-field for given V_i .

The constraint matrix K for finding 'W' is given by $K = \left[\frac{dz}{du} ; \frac{dz}{dv}\right]^T$ and the normalization factor $Q = (WK+1)^{-1}$

2.4 Nonlinear balance equation as a dynamical constraint

Nonlinear balance equation is incorporated in the analysis as an indirect relationship between wind field and mass field. As a part of the analysis procedure, it partially covers the role of initialization. The model system is very sensitive to the state of initial balance between wind and pressure fields. Normally, the Objective Analysis used for deriving wind and pressure fields often show large imbalance between coriolis force and pressure gradient force which lead to large amplitude inertiogravity waves if the models are integrated using such initial data. Thus a state of balance between wind and pressure field is a basic prerequisite of initial data. Geostrophic relation is the simplest relation between wind and pressure field. However Charney (1955) showed that use of geostrophic relations for initialization may result in noticeable noise and alternatively suggested the use of nonlinear balance equation to relate the initial wind and pressure fields. The balance equation has been widely used for static initialization in one level and multilevel model equations. For Indian region, the static initialization has been successfully applied for a one level model equation by Ramanathan and Saha (1972).

3. DATA, SYNOPTIC SITUATIONS AND COMPUTATIONS

In this study daily data of winds and heights for a period of monsoon season of 3rd, 4th and 5th August 1988 from radiosonde observations over India and neighbourhood were utilised. The circulation system studied was a depression which formed in the head Bay of Bengal intensified and moved west of northwestward direction. Thus, we have a different synoptic situation every day with the movement of the monsoon system. The tracks of depression are shown in Fig.(1). The wind and height observations at 850 mb over the region from 1.875°N to 39.315°N and from 41.25°E to 108.75°E were collected and processed. The climatological normal wind and height for the month of August at the grid points were used as initial guess for the analysis.

The weight corresponding to u and v components of wind at 850 mb level were computed by $W = BK^T (O + F)^{-1}$. As in OI scheme we modelled BK^T by a continuous covariance function. The constraint matrix K for both u and v can be obtained by finding Jacobian matrix of z field with reference to the components. The normalization function which represent the local data density function at each grid point can be approximated by the reciprocal of sum of the weights at each grid point using the formula $Q=(WK+1)^{-1}$ for both u and v. They are given by diagonal matrix. Iteration output for (u,v) can be obtained from the successive correction separately for u and v.

In order to verify the performance of the scheme the analysis of wind field for different synoptic situations were made for detailed study. Because the depression is provided different synoptic situations every day - RMS errors are computed comparing with station observations. We have taken three iteration for different days. The iteration results and RMS errors are shown in Fig.(2), (3),(4),(5) and Table 1 respectively.

4. RESULTS AND DISCUSSIONS ON ANALYSIS OF WIND FIELD

The analyses of wind field after the first iteration and third iteration for 3,4 and 5 August have been examined. Table 1 showing the RMS errors for 3,4 and 5 Aug. 1985 suggests that the R.M.S. errors have not reduced with the iterations, in other words, improvement is not so much. However, the flow patterns seem to have been depicted well in both cases. The cyclonic circulation associated with the monsoon depression has shifted a little in both cases. The winds over Arabian sea have strengthened slightly. However, the improvement in the analysis if any, with the iterations, would be known only when these wind fields are used in the forecast model and the forecast fields are obtained and examined. In this study the model has not been integrated using these wind fields and hence it cannot be inferred with certainty that there have been improvement. It is felt that this preliminary study would be a forerunner for other detailed study, to be undertaken, particularly using the radiative data in the temperature analysis, wherein one has to consider nonlinear relations.

5. Concluding remarks

The analysis formulation and its verification presented in this study demonstrate the importance of nonlinear analysis scheme as a data assimilation tool. We have applied the Lorenc’s scheme for tropical region for the analysis of wind field by taking nonlinear balance equation as constraint. The result showed that the analyses are as good as those obtained using linear OI scheme. The wind field, which is of course more important in the tropics, is depicted well in the scheme. The study indicates that analyses results are encouraging but further experiments are necessary. In our study we have utilised the analysis correction method for wind analysis from observed height field through nonlinear balance equations. It is found that they depicted circulation features fairly well. The centres of depression agreed with the analyses from India Meteorological Department as well as with the observed track of depression. As compared to linear scheme no remarkable improvement is there in the analysis. One of the main advantage of Bayesian method is that they can handle nongaussian statistics. To maximise the utilisation of available observations, more effective method will be needed to incorporate the time dimension in the analysis algorithm. The study indicates that the scheme could be implemented in the analysis and has the potential to improve forecasts in the tropical region.

6. References

Bratseth, A.M.(1986) : Statistical interpolation by means of Successive Corrections.Tellus,38A,439-447.

Courtier, P., Talgrand, O. (1975) : Variational assimilation of meteorological observations with the adjoint vorticity equations, QJRMS, 113, 1331-1350.

Charney, J., Halm, M., Jastrow, R. (1969) : Use of incomplete historical data to infer the present state of the atmosphere. J.Atmos.Sci.,26,l160-1163.

Eyre, J.R. (1989) : Inversion of cloudy satellite sounding radiances by non linear optimum estimation. QJRMS, 115, 1001-1026.

Gandin, L.S. (1963) : The objective analysis of Meteorological fields, Leningrad, Gridmeteorogicheskoe, Jerusalem (1965).

Gill, P.E., Murrary,W., Wright, M.H. (1982) : Practical optimization; Academic press, London.

Hollingsworth, A. (1986 : Objective analysis for numerlcal weather prediction - "IUGG NWP Symposium, Meteorological Society of Japan, 11-59.

Lorenc, A.C. (1981) : A global 3-dimensional multivariate statistical analysis scheme. Mon.Wea.Review, 109, 701-721.

Lorenc, A.C. (1986) : Analysis methods for numerical weather prediction, JRMS, 112, 1177-1194.

Lorenc, A.C. (1988a) : Optimal nonlinear objective analysis, QJRMS, 114, 205-240.

Lorenc, A.C. (1991) : The meteorological office analysis correction data assimilation scheme, QJRMS (1991), 113, 59-89.

Mcpherson, R.D. (1975) : Program, problems and prospects in meteorological data assimilation. Bulletin of American Meteorological Society, 56, 1154-1166.

Rajamani, S., D.R. Talwalkar, S.P. Ray and P.U. Upasani (1983) : Objective analysis of wind field over Indian region by OI method, Mausam, 34, 43-50.

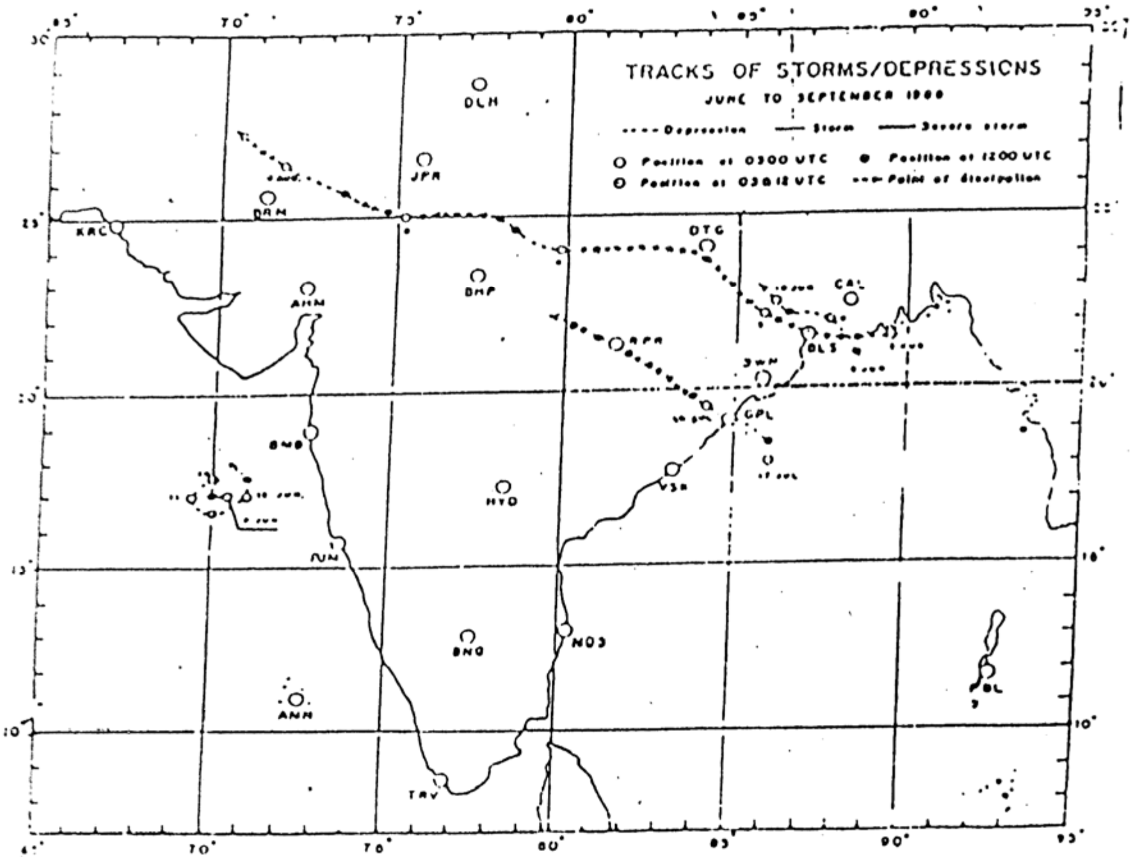
Ramanathan Y, Saha, K.R.7 (1972) : Application of a primitive equation barotropic model to predict movement of western disturbances. J. Applied Meteorology, 11, 268-272.

Acknowledgement

The authors are very grateful to Dr. N. Sen Roy, Director General of Meteorology, for his interest in the study and are thankful to Dr. R.R. Kelkar, ADGM, Dr. U.S. De, DDGM and Shri S.K. Shaha, Director, for their constant encouragement. They also thank Dr. R.N. Keshavamurty, Director, Indian Institute of Tropical Meteorology, Pune, for his valuable suggestions. They thank their colleagues Miss P.L. Kulkarni and S.G.Narkhedkar for their help in executing the computer codes.

Date	RMS Error 1st Iteration		RMS Error 3rd Iteration	
	u	v	u	v
03.08.1988	5.3	5.1	3.3	3.0
04.08.1988	4.3	4.1	6.2	5.3
05.08.1988	2.1	3.1	2.8	4.4

Table 1 RMS ERRORS IN MPS FOR WIND ANALYSIS COMPARING WITH STATION OBSERVATIONS 850 MB, AUGUST 1988, 12 GMT



FIG(1) TRACKS OF MONSOON DEPRESSION - 1988

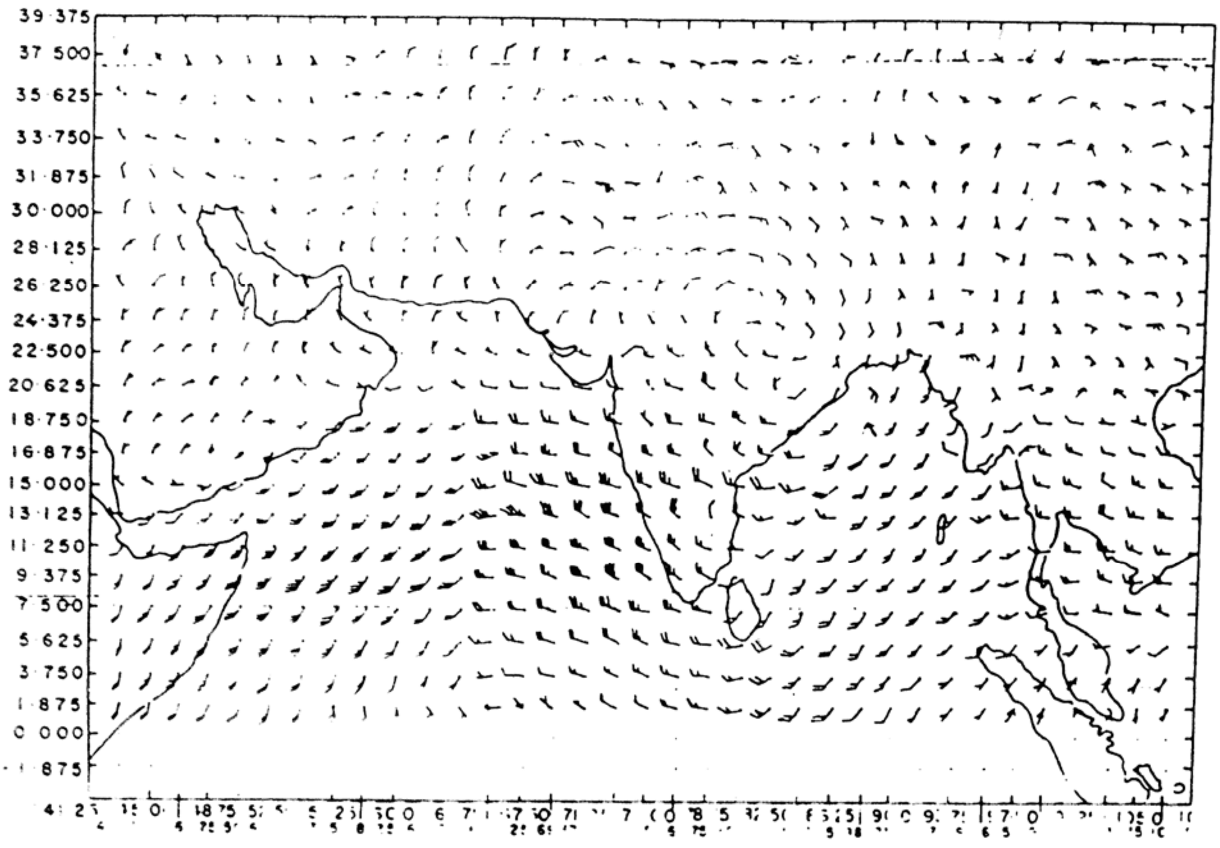


Fig.(2) : Objectively analysed wind field (Non linear interpolation scheme). 3rd iteration. 3rd August 1988, 12 GMT.

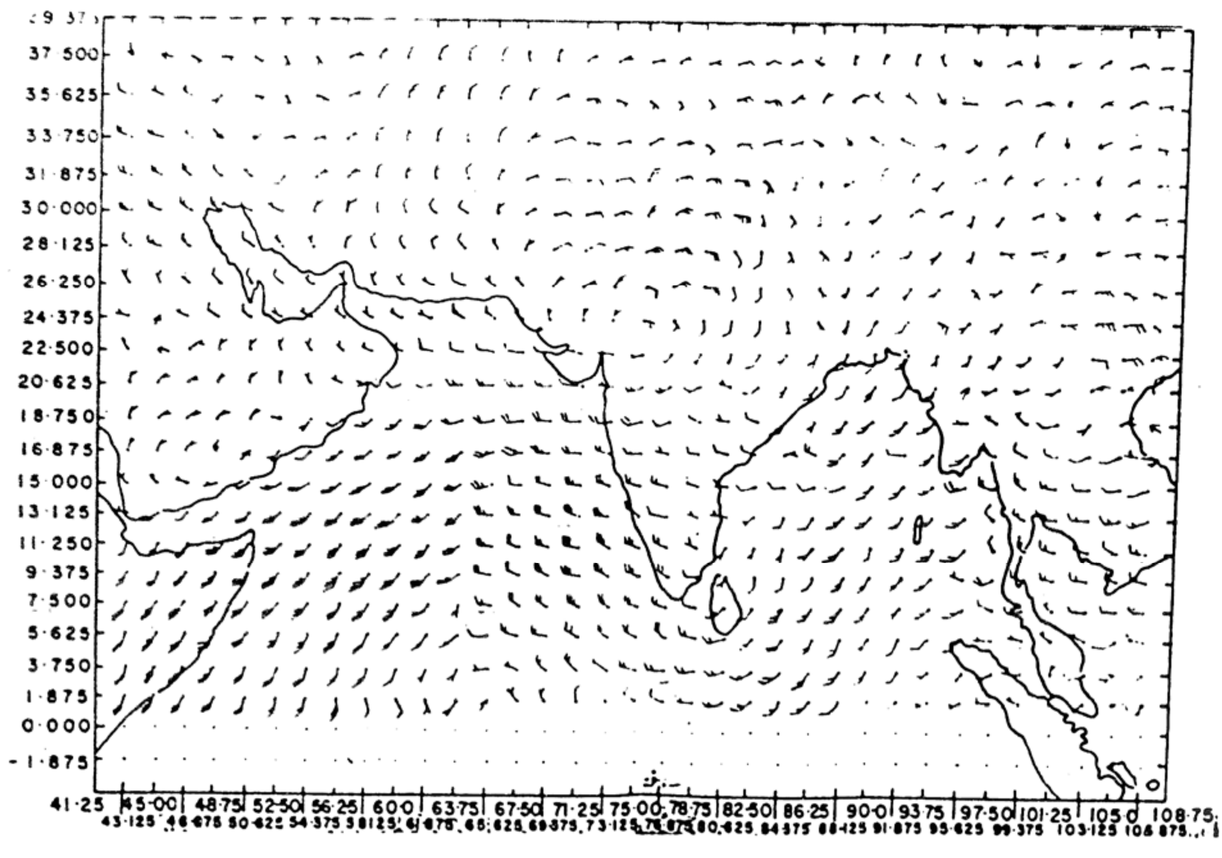


Fig.(3) : Objectively analysed wind field (Non linear interpolation scheme). 3rd iteration. 5th August 1988; 12 GMT.

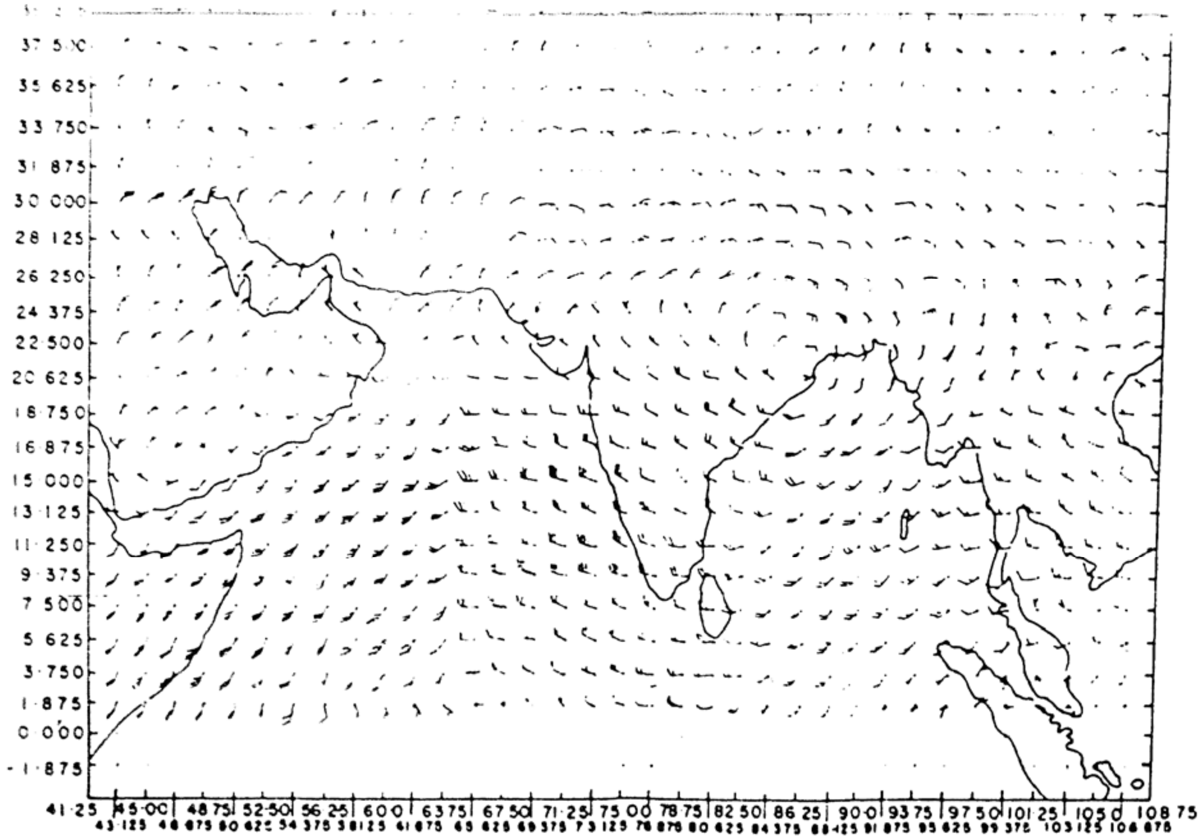


Fig.(4) : Objectively analysed wind field (Non linear interpolation scheme). 1st iteration. 3rd August 1988; 12 GMT.

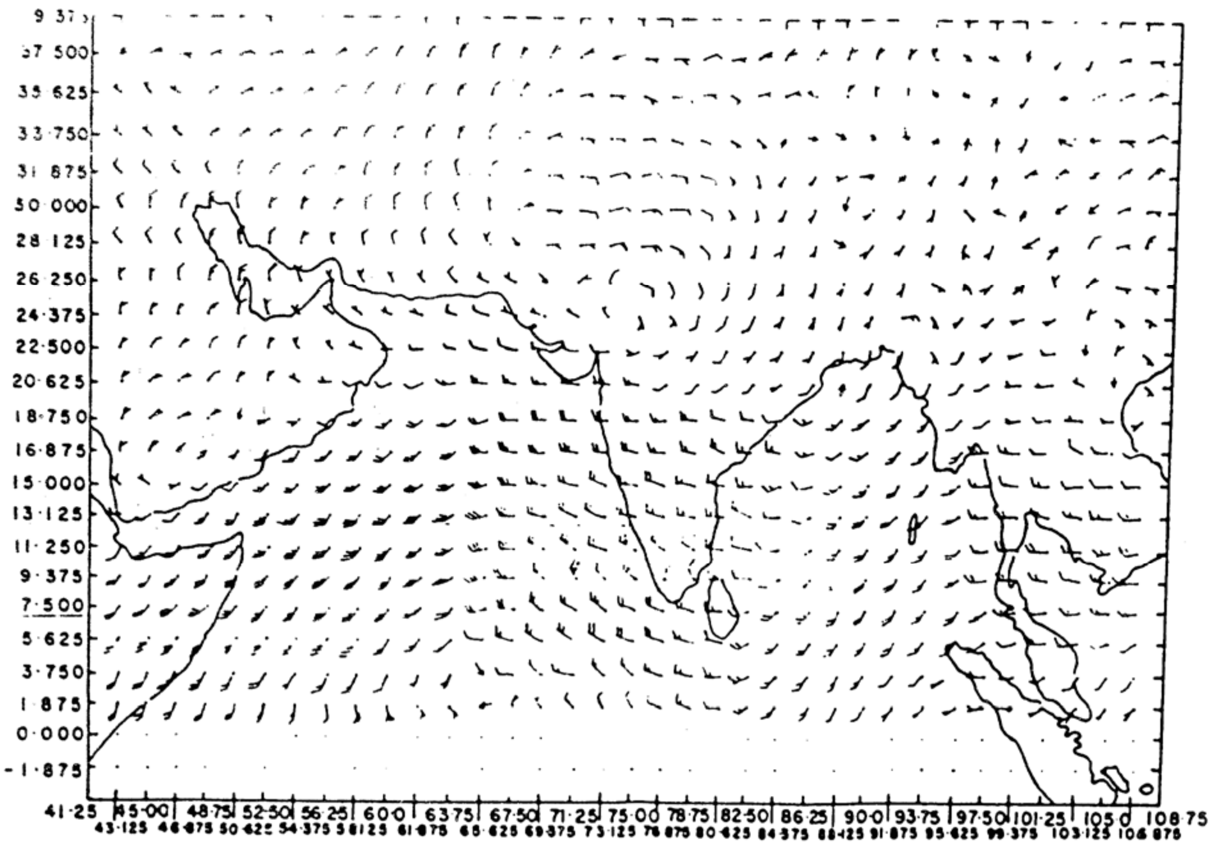


Fig.(5) : Objectively analysed wind field (Non linear interpolation scheme). 1st iteration. 5th August 1988; 12 GMT.