

# IMPROVED COMPUTING EFFICIENCY OF CROSS-CORRELATION IN THE FOURIER DOMAIN

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## ABSTRACT

Calculation of the cross-correlation can be carried out in both the spatial and Fourier domain. The relative efficiency of the two techniques is dependent on the search area and target area sizes. The Fourier Transform method becomes more efficient as the target area size approaches the search area and with larger search and target areas. This paper outlines techniques which have been investigated into improving the run-time performance of cross-correlation in the Fourier domain.

Implementations of cross-correlation in the Fourier Domain generally use Fast Fourier Transform (FFT) algorithms, which require the search and target data sets to be extended with zeros so that their sizes are a common power of 2. These algorithms are referred to as Radix-2 transforms, and work by combining transform levels at successively increasing powers of 2. A potentially more efficient FFT algorithm, called a Mixed Radix FFT, avoids the artificial increase of the data sets and consequently the number of calculations. The method works by calculating transforms at levels equal to the prime factors of the data set size and combining them at levels of the prime multiples. The algorithm's efficiency is heavily dependent on the highest prime factor, being more efficient the lower this factor is. The Mixed Radix algorithm has been compared to the Radix-2 algorithm for a number of search/target data set sizes in which the highest prime factors are 3 or 5 and significant improvements in run-time performance have been achieved. The results show that performance can be optimised by careful selection of the data set sizes. The paper also presents some results based on real imagery data, comparing the displacement vectors derived with a traditional spatial cross-correlation technique to those derived with the fast Mixed Radix FFT.

## 1. INTRODUCTION

The Meteorological Product Extraction Facility (MPEF) is being developed as part of the Meteosat Second Generation (MSG) Ground Segment. There is an on-going requirement to minimise computing processing power costs for the MSG MPEF. The Atmospheric Motion Vector (AMV) product to be generated in the MSG MPEF is a significant drain on computing resources, and in particular the Cross-correlation target matching technique poses the highest CPU load. Hence, investigations have been carried out to optimise the efficiency of cross-correlation. The initial results of this work are presented in this paper.

Section 1 is this introduction. Section 2 places cross-correlation in the context of generally used operational target matching techniques. Section 3 briefly summarises the traditional approach to cross-correlation, which can be calculated either in the spatial or Fourier domain. A new approach to calculating cross-correlation in the Fourier domain is introduced and the criteria for its optimal

performance are discussed in respect to the target and search area sizes used in the MPEF environment. In section 4, investigations of the new Fourier domain technique have concentrated on validation against the currently used Fourier domain technique, using synthetic data; and validation against the spatial domain technique using data generated in the MPEF environment. Section 5 describes the results of performance evaluations, comparing the new Fourier domain technique against the current Fourier and spatial domain techniques. Section 6 assesses the conclusions of the work carried out so far and provides recommendations for further investigations.

## 2. CURRENT OPERATIONAL MATCHING TECHNIQUES

Cross-correlation is one of the standard statistical techniques used for target matching (e.g. Tokuno, 1996). It is particularly suitable for small translations, rotations or scale changes. Cross-correlation can be carried out in either the spatial or Fourier domain. While being a rigorous technique, it is computationally intensive. To minimise computation, various techniques have been adopted which reduce the number of correlation surface points calculated. These include for example initially calculating the correlation surface at defined intervals, then concentrating on areas in which either the correlation value is above a threshold, or is higher than in other areas (e.g. Xu and Zhang, 1996, Bhatia et al., 1996). The sampling interval is then iteratively reduced as the number of sub-areas within the correlation surface are reduced. While being more computationally efficient, the accuracy of the results are not guaranteed and a steep correlation peak may be missed if a sampling interval is too large.

Other matching techniques have been used which are less computationally intensive. The most common is the Sum of Squared Distances (SSD) (e.g. Velden et al., 1996) or Euclidean distance. A variation of this is the Sum of Absolute Value of Difference (SAVD). The SSD method is used in the MPEF environment as a complementary method to cross-correlation.

The Sequential Similarity Detection Algorithm (SSDA) was developed primarily to minimise computing time. For each correlation surface point, each of the contributing target/search area pixels are selected in turn at random. The SAVD for this correlation surface point is incremented accordingly. The process stops when a user defined SAVD threshold is reached and the running total of contributions is recorded. The recorded running total will be different for each correlation surface point and the correlation peak is assigned to the point at which the running total is highest. However, selection of the threshold value is crucial and needs to be very close to the expected value for the peak.

Cross-correlation is generally recognised as an optimum matching technique and has been shown to be effective in the presence of noisy data.

## 3. THE NEW APPROACH

### 3.1 Background

The standard formula for cross-correlation between two variables T and S is given by:

$$R_{ij} = \sum_{m=1}^{N_T} \sum_{n=1}^{N_T} T_{i,j} S_{i+m,j+n} \quad (1)$$

The target area is denoted by T and the search area by S. For a square target size, with side length  $N_T$ , the total number of pixels used to compute one correlation value is  $N = N_T^2$ . If the pixels within the target area are identified by (m,n) and the target location within the search area

by (i,j), such that the target is always fully contained within the search area, then  $T_{mn}$  and  $S_{i+m,j+n}$  uniquely identify pixel count values within the target and search areas.

The cross-correlation can be normalised to prevent false correlation peaks arising from changes in the search area local means, and any local additive bias differences can also be removed. The expression for the cross-correlation is expanded to produce the cross-correlation coefficient defined by:

$$r_{ij} = \frac{\sum_{m=1}^{N_T} \sum_{n=1}^{N_T} (T_{mn} - \mu_T)(S_{i+m,j+n} - \mu_{Sij})}{\left[ \sum_{m=1}^{N_T} \sum_{n=1}^{N_T} (T_{mn} - \mu_T)^2 \right]^{1/2} \left[ \sum_{m=1}^{N_T} \sum_{n=1}^{N_T} (S_{i+m,j+n} - \mu_{Sij})^2 \right]^{1/2}} \quad (2)$$

where

$$\mu_T = \frac{1}{N_T} \sum_{m=1}^{N_T} \sum_{n=1}^{N_T} T_{mn} \quad \text{and} \quad \mu_{Sij} = \frac{1}{N_T} \sum_{m=1}^{N_T} \sum_{n=1}^{N_T} S_{i+m,j+n}$$

This is the classical expression for cross-correlation in which all target and search pixels contribute to the correlation.

The driving factor in determining the computing efficiency of the cross-correlation coefficient expression in equation 2 is the cross-correlation term in equation 1. In efforts to optimise this efficiency, use has been made of a property of the Correlation Theorem which states that the Fourier Transform of the correlation of two images is the product of the Fourier transform of one image and the complex conjugate of the Fourier transform of the other. The cross-correlation can hence be implemented in the Fourier Domain by three Fourier transforms, and all elements of equation 1 can be expressed by:

$$R_{ij} = \left[ F^{-1} \{ F(S) F^*(T) \} \right]_{ij} \quad (3)$$

### 3.2 FFT Methods

Traditional implementations of cross-correlation in the Fourier Domain generally use FFT algorithms. The FFT algorithm is an efficient method for computing the n-point transformation defined by:

$$\alpha_k = \sum_{j=0}^{n-1} x_j \exp(i2\pi jk/n) \quad (4)$$

for  $k=0,1,\dots,n-1$ , where  $\{x_j\}$  and  $\{\alpha_k\}$  are both complex valued. The basic idea of many implementations of the current form of the FFT is that developed by Cooley and Tukey (1965). It involves that of factoring the data set size  $n$ ,

$$n = \prod_{i=1}^m n_i \quad (5)$$

and then decomposing the transform into  $m$  steps with  $n/n_i$  transformations of size  $n_i$  within each step. Most implementations of the FFT have concentrated on the special case of  $n = 2^m$ , the

Radix-2 FFT, which works by calculating and combining transforms of size 2 at successively increasing levels of powers of 2. The transform is decomposed into  $\log_2 n$  steps with  $n/2$  transformations of size 2 within each step. The Radix-2 FFT is generally easier to implement, is very efficient because of the low transform size, and the restricted choice of values of  $n$  is adequate for the majority of applications. The Radix-2 FFT algorithm can be computed in  $O(n \log_2 n)$  operations, which is considerably faster than the  $O(n^2)$  operations for the ordinary Fourier Transform.

In comparing the spatial and Fourier domain methods, for a target size of  $N_T$  and search size of  $N_S > N_T$ , the Radix-2 FFT method becomes relatively more efficient as  $N_T$  approaches  $N_S$  and more efficient than the spatial method with larger  $N_T$  and  $N_S$ . For Radix-2 FFT the data set size should be equal to a power of 2, equally for both target and search sizes (so that the frequency samples are identical). This implies that the available data sets are necessarily padded with zeros to take their sizes to the nearest power of 2. However, this method is still relatively more efficient than the spatial method, despite the large number of redundant calculations due to the padded zeros.

Using the principle of equation 5, other FFT methods have been adopted which subdivide transforms down to levels of 4, 8, and 16 for example, respectively called Radix-4, Radix-8 and Radix-16. These can take advantage of special symmetries of that level, and can be faster than the Radix-2 method under some circumstances but only by up to 20 or 30 per cent (Press et al., 1992). However, these methods still require data set sizes of powers of 2.

### 3.3 Mixed Radix Method

A potentially more efficient FFT method is proposed in this paper, which minimises the size of the data set and potentially the number of calculations. This is called the Mixed Radix FFT. As for the Radix-2, 4, 8, 16 methods, the starting point is equation 5. The algorithm works by decomposing the data set size into its prime factors. The FFT is built up by calculating and combining transforms at levels equal to the prime factors of the data set size. The transform is decomposed into a number of steps equal to the number of prime factors. For each prime factor,  $p$  there are  $n/p$  transforms of dimension  $p$ . Singleton (1965) provides an efficient implementation of the Mixed Radix method by optimising computation of transform steps for odd factors and some extensions for the two-dimensional case can be found in Press (1992).

The efficiency of the Mixed Radix method relative to Radix-2 is mainly dependent on two variables

- the relative sizes of the data sets needed for the 2 methods, and
- the highest prime factor of the transform data set size.

The Mixed Radix method allows the transform data set size to be reduced for both target and search area from the nearest power of 2 above  $N_S$  to as low as  $N_S$ . Furthermore, the lower the highest prime factor of the transform data set size is, the relatively more efficient is the Mixed Radix method. Singleton (1965) indicates that the Mixed Radix FFT performance is optimised for highest prime factors up to 5. The performance advantage over Radix-2 would be expected to reduce as the highest prime factor increased, but would also be dependent on the relative transform data set sizes.

## 4. VALIDATION

### 4.1 Theoretical Validation

The theoretical validation of the Mixed Radix algorithm was carried out using simulated search and target data. The basic metric used to validate the algorithm was to compare the output correlation surface (as defined in equation 2) for the Mixed Radix and Radix-2 cases. Both Mixed Radix and Radix-2 FFT algorithms were tested on identical input data. The correlation value at each point on the surface was compared for the 2 cases and the RMS difference calculated. In addition, the maximum difference at any one point on the surface was also recorded. Less rigorously, viewing the graphical display of the respective correlation surfaces allowed an easy non-mathematical way to compare the methods.

#### 4.1.1 Description of Test Variables

Three target /search area combinations were tested, representing the combinations applicable in the MPEF environment. The target area consists of a circular tracer, each count value within the tracer being identical. The tracer diameter is 24 pixels for the 24/80 and 32/96 target/search area combinations and 12 pixels for the 16/72 combination. The target image is replicated over the central search location of the search area, so that the correlation peak is at the centre of the correlation surface.

The correlation surface, over which the RMS difference is calculated, is the set of cross-correlation coefficients described in equation 2. The total number of points on the correlation surface is dependent on the target area and search area size and is given by  $(\text{search area size} - \text{target area size} + 1)^2$ . Note the number of correlation surface points is identical for the 16/72 and 24/80 target/search area combinations.

#### 4.1.2 Results and Discussion

Table 1 lists the RMS and maximum difference between Mixed Radix and Radix-2 FFT methods over the correlation surface samples.

Table 1 - Relative Accuracy of Mixed Radix and Radix-2 FFT methods over Correlation Surface

	Correlation Surface: Mixed Radix vs. Radix-2 FFT Methods	
	RMS Difference	Maximum Difference
Target Area 16x16 Search Area 72x72	$3.81 \times 10^{-6}$	$2.26 \times 10^{-5}$
Target Area 24x24 Search Area 80x80	$3.91 \times 10^{-7}$	$1.50 \times 10^{-6}$
Target Area 32x32 Search Area 96x96	$2.02 \times 10^{-6}$	$1.64 \times 10^{-5}$

By definition the cross-correlation coefficient lower and upper limits are -1 and 1 respectively. Table 1 shows that the RMS and maximum differences are less than  $10^{-4}$  for all target/search area combinations. These low orders of magnitude suggest that the differences in accuracy of the Radix-2 and Mixed Radix FFT methods are statistically insignificant for the synthetic data used in these tests. Analysing the graphical displays of all the correlation surfaces further supports this view.

Section 4.2 introduces a further level of rigour to the validation procedures by assessing the accuracy of the Mixed Radix method in the MPEF environment using 'real' data.

## 4.2 Validation in the MPEF Environment

### 4.2.1 Criteria for Analysis

Preliminary investigations have been carried out into validating the Mixed Radix FFT method against the spatial domain technique in the MPEF environment. A series of wind vectors were generated over the Earth's globe for the MPEF Water Vapour channel for both techniques using identical image data. A statistical analysis of the differences is highlighted in two histogram representations which respectively concentrate on separating the direction differences (degrees) and the speed differences (m/s) of the 3280 generated wind vectors into classes. The statistics are also presented for three quality indices - all vectors, vectors with a quality index above 0.3, and vectors with a quality index above 0.6. A summary statistical table lists the number of vectors, speed bias, mean vector difference, RMS difference, mean speed and normalised RMS (RMS difference/mean speed).

### 4.2.2 Results and Discussion

Figure 1 provides a high level view of all the wind vectors. The Mixed Radix FFT vectors are plotted in yellow and the spatial domain vectors are overlaid in blue. Hence, where there is very good correspondence, the FFT vectors are obscured by those produced by the spatial domain technique. From the high level view it can be seen that in most areas there is good agreement.

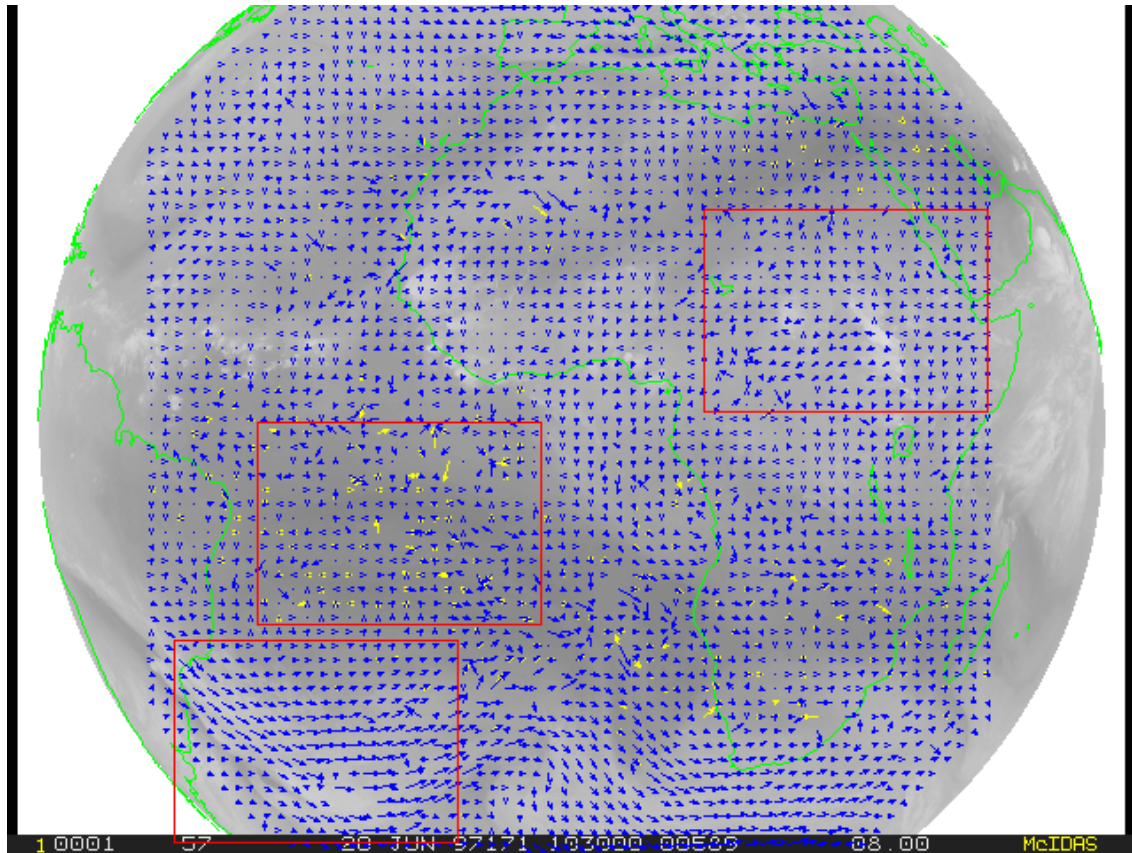


Figure 1 - MPEF WV Channel Wind Vectors (High Level View) (Yellow: Mixed Radix FFT Blue: Spatial Domain)

Figure 2 expands an area of larger discrepancies highlighted in Figure 1 to illustrate differences between the two wind vector sets.

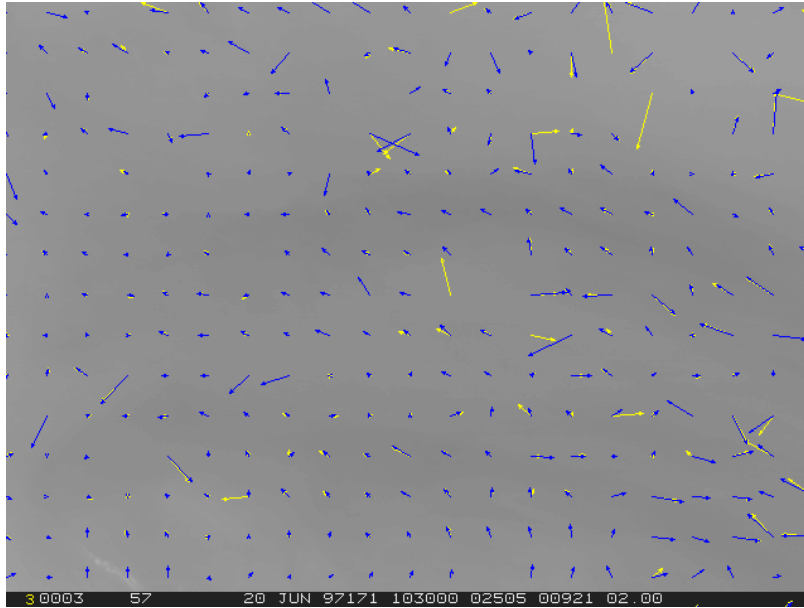


Figure 2 - MPEF WV Channel Wind Vectors (Low Level View) (Yellow: Mixed Radix FFT Blue: Spatial Domain)

Figures 1 and 2 indicate that the discrepancies in the methods may be confined to isolated areas, and it may be that in these isolated areas, cross-correlation is not the optimal matching technique (e.g. if the target contrast is poor).

Figures 3 and 4 and Table 2 detail the statistics, which show that in over 80% of cases there is less than 1.0 degree direction difference and 0.1 m/s speed difference. As the quality index threshold rises, the two methods further converge, with up to approximately 85% of wind vectors with differences in the lowest classes. Table 2 shows that the mean vector difference, RMS vector difference and Normalised RMS Difference decrease as the quality threshold increases.

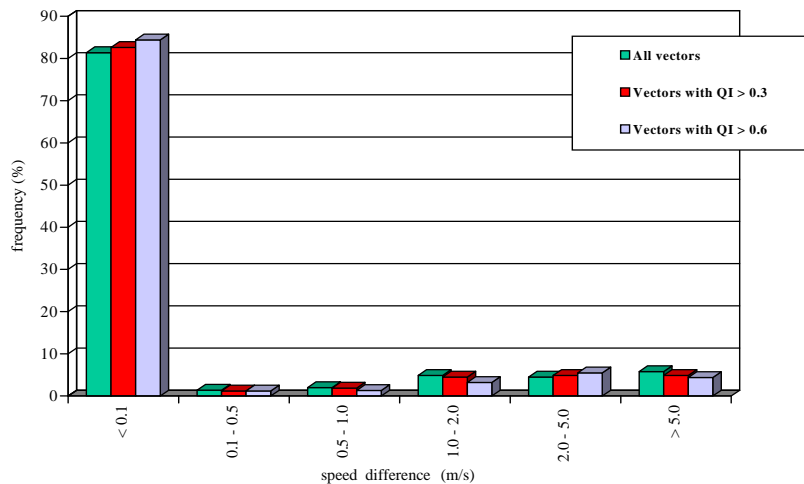


Figure 3. Frequency of vectors in speed difference classes (m/s). Total number of vectors is 3280

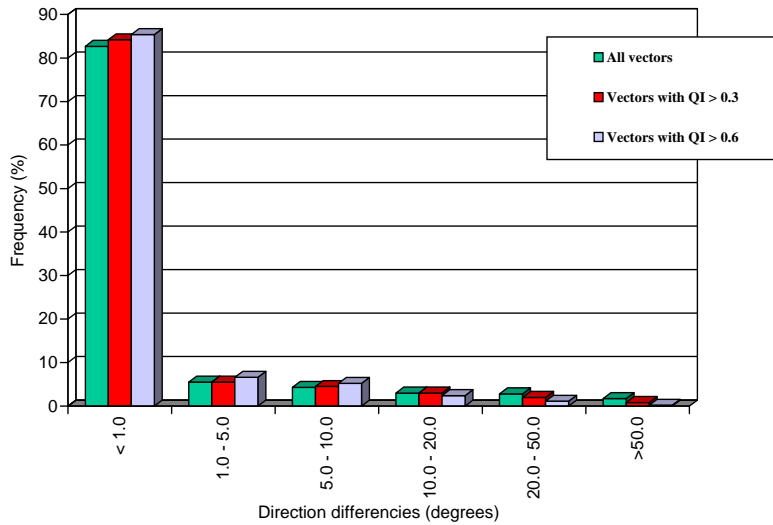


Figure 4. Frequency of vectors in direction difference classes (degrees)

Table 2. Vector Difference Statistics

	No of vectors	Speed bias	Mean vecdiff	RMS diff	Mean speed	NRMS
All	3280	0.0	1.1	4.9	17.9	0.27
QI>0.3	2639	0.0	0.9	3.9	17.7	0.22
QI>0.6	1491	-0.1	0.8	3.1	20.6	0.15

## 5. PERFORMANCE

### 5.1 Environment

The performance tests were carried out on a Hewlett-Packard HP 9000 Series Model C180-XP machine (single processor), with 180 MHz CPU, 128 MB Memory, and 1 MBI and 1 MBD off-chip Primary Cache, and (optimum) 720 MFLOPS.

Performance was evaluated for both Radix-2 and Mixed Radix FFT methods and the spatial domain method. All methods have been optimised for maximum efficiency on the Hewlett-Packard machine. The harness programs and algorithms were written in FORTRAN 77. Different compiler optimisations were tested and it was found that generally the +O3 level provided optimum performance for both spatial and FFT methods.

To assess relative performance, the nature of the input data is not important. The performance was computed using the synthetic data described in Section 4.1.

The performance test run times necessarily include time spent calculating non-FFT components of the cross-correlation coefficient of equation 2. The normalisation terms in this expression have also been computationally optimised. The test harness produced to analyse the performance enables 2000 segments to be processed, i.e. 2000 correlation surfaces are computed. This means that the CPU time required to process the data is in sufficiently measurable units to effectively analyse comparisons in performance.

### 5.2 Results and Evaluation

In replicating each test, investigations showed that there was a discrepancy of less than 2% between any one individual time and the average. The tests were also carried out when no other



significant processes were being run on the host machine and hence the real times were comparable to the CPU times. The relative amount of CPU time spent in I/O operations was also insignificant, confirming that effectively all the CPU time was spent in computing the correlation surface. Table 3 lists the CPU times for all tests.

Table 3 - CPU Run Time performance for Spatial and FFT Methods

	CPU Run Time (min:secs)		
	Spatial	Radix-2 FFT	Mixed Radix FFT
Target Area 16x16 Search Area 72x72	0:16.5	0:26.6	0:10.2
Target Area 24x24 Search Area 80x80	0:32.8	0:26.7	0:12.9
Target Area 32x32 Search Area 96x96	1:26.9	0:28.1	0:16.3

The above table shows that the Mixed Radix FFT is significantly more efficient than Radix-2 for all data sets investigated. The Mixed Radix method achieves run times between approximately 40 and 60 % of Radix-2. This is due to the relatively lower data set sizes needed for the Mixed Radix FFT and also the fact that the highest prime factors for all data sets are relatively low. Analysing Table 3, for the Radix-2 FFT method the data set size is 128 X 128 for both target and search area sizes, for all 3 target/search area combinations. The FFT run time contribution is the same for these 3 cases; minor differences in run time for these three cases are due to the non-FFT component contributions which vary with the data set size. The relatively small differences indicate that the non-FFT components do not make a significant contribution to the run-time. For the Mixed Radix method the data set size is significantly lower than Radix-2 and the relative improvement in performance is greater when the data set size is lower, as illustrated by the 32/96 and 16/72 target/search combinations, in which the highest prime factor (3) is the same. The 24/80 target/search combination data set size has a higher prime factor (5), but this does not appear to degrade performance.

The use of a Mixed Radix FFT ensures that run time performance is significantly better than for the corresponding spatial domain method, for all three target/search combinations. The Mixed Radix FFT achieves run times between approximately 20 and 60 % of the spatial domain method. The relative performance improves as the data set increases.

## 6. CONCLUSIONS AND RECOMMENDATIONS

This paper has analysed a potentially improved method for computing cross-correlation in the Fourier domain. This has been driven by the requirement to reduce computing processing power costs for the MSG MPEF. The cross-correlation computation within the Atmospheric Motion Vectors (AMV) product uses a significant percentage of computing time and hence is a prime area to address optimisations in run-time performance, leading to potentially reduced computing costs.

The basis of the method is the use of a Mixed Radix FFT algorithm rather than the more traditional Radix-2 method. The use of a Mixed Radix FFT avoids artificially increasing the data set size to the nearest power of 2, however its performance is crucially determined by the value of the highest prime factor of the data set size. The investigations presented in this paper have concentrated on 2 areas : comparing the accuracy of the Radix-2, Mixed Radix and spatial domain methods both in a synthetic and real (MPEF) environment, and comparing the run-time performance of the algorithms.

Performance investigations have shown that the Mixed Radix FFT method is significantly more efficient than Radix-2 for all target/search area sizes. Differences in accuracy between the two

methods are negligible. The Mixed Radix FFT is also significantly more efficient than the spatial domain method, being relatively more efficient as the data set increases.

In considering the relative performance merits of the Mixed Radix, Radix-2 FFT's and spatial domain methods for applications outside MSG MPEF's immediate area of interest, consideration of the relative data set sizes and the highest prime factor are the driving factors. For MSG MPEF applications, the data set size is significantly reduced by using the Mixed Radix FFT method, and the highest prime factor is very low (3 for two target/search area combinations and 5 for the other). If the highest prime factor were to be significantly high, for example if the data set size is 82 (highest prime factor 41) then the method would be expected to be very inefficient. In circumstances similar to this, it would be prudent to artificially increase the data set size to reduce the highest prime factor.

The results using real water vapour imagery data indicates a strong correlation between the Mixed-Radix and the spatial domain correlation. A much larger wind vector data set is needed to finalise the validation, and it is recommended that further data sets, including wind vectors generated for a wider range of channels and target/search area sizes be investigated. Differences between the spatial and Fourier domain methods are highlighted in the presence of noisy data or in areas with small contrast for which the cross-correlation might not be the optimum technique. It is recommended that to assist in the validation, for areas in which the discrepancy is highest, the cross-correlation technique be compared against the Euclidean Distance technique.

The case for using the Mixed Radix FFT technique as opposed to the spatial domain has been demonstrated from the performance point of view in this paper. However, more investigations are required to fully address the relative wind vector accuracy produced by the spatial and Fourier techniques in the MPEF environment. Once these have been completed then a balanced prognosis can be made on the operational use of the new Fourier domain technique in the MPEF environment.

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